Promotion of cooperation induced by the interplay between structure and game dynamics

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Abstract
We consider the coupled dynamics of the adaption of network structure and the evolution of strategies played by individuals occupying the network vertices. We propose a computational model in which each agent plays a $n$-round Prisoner’s Dilemma game with its immediate neighbors, after that, based upon self-interest, partial individuals may punish their defective neighbors by dismissing the social tie to the one who defects the most times, meanwhile seek for a new partner at random from the neighbors of the punished agent. It is found that the promotion of cooperation is attributed to the entangled evolution of individual strategy and network structure. Moreover, we show that the emerging social networks exhibit high heterogeneity and disassortative mixing pattern. For a given average connectivity of the population and the number of rounds, there is a critical value for the fraction of individuals adapting their social interactions, above which cooperators wipe out defectors. Besides, the effects of the average degree, the number of rounds, and the intensity of selection are investigated by extensive numerical simulations. Our results to some extent reflect the underlying mechanism promoting cooperation.

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1. Introduction

Cooperative behaviors are ubiquitous in real-world, ranging from biological systems to socioeconomic systems. However, the question of how natural selection can lead to cooperation has fascinated evolutionary biologists for several decades. Fortunately, together with classic game theory, evolutionary game theory provides a systematic and convenient framework for understanding the emergence and maintenance of cooperative behaviors among selfish individuals \cite{1,2}. Especially, the Prisoner’s Dilemma game (PDG) as a general metaphor for studying the evolution of cooperation has attracted considerable interests \cite{3}.

In the original PDG, two players simultaneously decide whether to cooperate (C) or to defect (D). They both receive $R$ upon mutual cooperation and $P$ upon mutual defection. A defector exploiting a C player gets...
$T$, and the exploited cooperator receives $S$, such that $T > R > P > S$ and $2R > T + S$. As a result, it is best to defect regardless of the co-player’s decision. Thus, in well-mixed infinite populations, defection is the evolutionarily stable strategy (ESS), even though all individuals would be better off if they cooperated. Thereby this creates the social dilemma, because when everybody defects, the mean population payoff is lower than that when everybody cooperates. In a recent review Nowak suggested five rules for the evolution of cooperation (see Ref. [4] and references therein). Most noteworthy, departure from the well-mixed population scenario, the rule “network reciprocity” conditions the emergence of cooperation among players occupying the network vertices [5]. That is, the benefit-to-cost ratio must exceed the average number of neighbors per individual. Actually, the successful development of network science provides a convenient framework to describe the population structure on which the evolution of cooperation is studied. The vertices represent players, while the edges denote links between players in terms of game dynamical interactions. Furthermore, interactions in real-world network of contacts are heterogeneous, often associated with scale-free (power-law) dependence on the degree distribution, $P(k) \sim k^{-\gamma}$ with $2 < \gamma < 3$. Accordingly, the evolution of cooperation on model networks with features such as lattices [6–9], small-world [10–12], scale-free [13–15], and community structure [16] has been scrutinized. Interestingly, Santos et al. found that scale-free networks provide a unifying framework for the emergency of cooperation [13].

From the best of our knowledge, so far much previous works of games on networks are based on crystallized (static) networks, i.e., the social networks on which the evolution of cooperation is studied are fixed from the outset and not affected by evolutionary dynamics on top of them. However, interaction networks in real-world are continuously evolving ones, rather than static graphs. Indeed, individuals have adaptations on the number, frequency, and duration of their social ties base upon some certain feedback mechanisms. Instead of investigating the evolutionary games on static networks which constitute just one snapshot of the real evolving ones, recently, some researchers proposed that the network structure may co-evolve with the evolutionary game dynamics [17–23]. Interestingly, as pointed out in Refs. [18,19,22], the entangled evolution of individual strategy and network structure constitutes a key mechanism for the sustainability of cooperation in social networks. Therefore, to understand the emergence of cooperative behavior in realistic situations (networks), one should combine strategy evolution with topological evolution. From this perspective, we propose a computational model in which both the adaptation of underlying network of interactions and the evolution of behavioral strategy are taken into account simultaneously. In our model, each agent plays a $n$-round PDG with its immediate neighbors, after that, based upon self-interest, partial individuals may punish their defective neighbors by dismissing the social tie to the one who defects the most times, meanwhile seek for a new partner at random from the neighbors of the punished agent. We shall show that such individual’s local adaptive interactions lead to the situation where cooperators become evolutionarily competitive due to the preference of assortative mixing between cooperators. The remainder of this paper is organized as follows. In the following section, the model is introduced in detail. Section 3 presents the simulation results and discussions. We finally draw conclusions in Section 4.

2. The model

We consider a symmetric two-player game where $N$ individuals engage in the PDG over a network. The total number of edges $M$ is fixed during the evolutionary process. Each individual $i$ plays with its immediate neighbors defined by the underlying network. The neighbor set of individual $i$ is denoted as $\Omega_i$, which is allowed to evolve according to the game results. Let us denote by $s_i$ the strategy of individual $i$. Player $i$ can follow two simple strategies: cooperation $[C, s_i = (1, 0)^T]$ and defection $[D, s_i = (0, 1)^T]$ in each round. Following previous studies [24,25], the payoff matrix $M$ has a rescaled form depending on a single parameter,

$$M = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix},$$

where $1 < b < 2$.

In each round, each agent plays the same strategy with all its neighbors, and accumulates the payoff, observing the aggregate payoff and strategy of its neighbors. The total income of the player at the site $x$ can be
expressed as
\[ P_x = \sum_{y \in O_x} s_x^T M s_y, \] (2)
where the sum runs over all the neighboring sites of \( x \), \( O_x \). In evolutionary games the players are allowed to adopt the strategies of their neighbors after each round. Then, the individual \( x \) randomly selects a neighbor \( y \) for possibly updating its strategy. The site \( x \) will adopt \( y \)'s strategy with probability determined by the total payoff difference between them [6,26,27]:
\[ W_{sx \rightarrow sy} = \frac{1}{1 + \exp\left(\frac{b(P_x - P_y)}{C_0}\right)/C_1}, \] (3)
where the parameter \( b \) is an inverse temperature in statistical physics, the value of which characterizes the intensity of selection. \( b \rightarrow 0 \) leads to neutral (random) drift whereas \( b \rightarrow \infty \) corresponds to the imitation dynamics where the \( y \)'s strategy replaces \( x \)'s whenever \( P_y > P_x \). For finite value of \( b \), the larger \( b \) is, the fitter strategy is more apt to replace to the less one, thus the value of \( b \) indicates the intensity of selection.

In the present model, we assume each agent plays a \( n \)-round PDG with its neighbors (\( n \geq 1 \)), and then \( m \) randomly selected individuals are allowed to adapt their social ties according to the game results (\( 1 \leq m \leq N \)). Here the individuals are endowed with the limited cognitive capacities—each agent records the strategies of its opponents used in the \( n \)-round game. Then they are able to decide to maintain those ties from which they benefit from, and to rewire the adverse links. For the sake of simplicity, if someone is picked for updating its neighbors, only the most disadvantageous edge is rewired. It dismisses the link to the one, who defects the most times (if there exist more than one individuals who defect the same maximum times, the one is chosen at random), and redirects the link to a random neighbor of the punished (see Fig. 1 as an illustrative example). The advantage of rewiring to neighbor's neighbor is twofold: first, individuals tend to interact with others that are close by in a social manner [28], i.e., friend's friend is more likely to become a friend (partner); second, every agent is seeking to attach to cooperators, thus redirecting to neighbor's neighbor will be a good choice since the neighbor also tries to establish links with cooperators. Hence rewiring to a neighbor of a defector is no doubt a good choice for individuals with local information only [22]. Herein, the parameters \( n \) and \( m \) can be viewed as the corresponding time scales of strategy evolution and network structure adaptation. As our strategy evolution uses synchronous updating, while evolution of network topology adopts asynchronous updating, in our case, the strategy updating event proceeds naturally much more frequent than evolution of network structure (as \( N \cdot n > m \)). Nevertheless, even though network structure adaption is much lower than game dynamics, cooperation is still promoted by the efficient interplay between the two dynamics.

Let us point out the differences between our model and previous works. In Refs. [18,19], the evolution of strategy adopted the “best-take-over” update rule where each agent imitates the strategy of the best neighbor. Besides, individuals are divided into two types based on the payoffs: satisfied and unsatisfied. If individual’s payoff is the highest among its neighbors, then it is satisfied. Otherwise, it is unsatisfied. The network

![Fig. 1. Illustration of individual's local adaptive interactions. Assuming A is picked for updating its social ties after playing n-round Prisoner's Dilemma game with immediate neighbors. A dismisses the link to B, who defects the most times, and rewires the link to C, a random neighbor of B.](image-url)
adaptation dynamics is restricted to the ones who are defectors and unsatisfied. Thus, the unsatisfied defector breaks the link with defective neighbor with probability $p$, replaces it by randomly choosing agent uniformly from the network. More recently, Ref. [22] proposed another minimal model that combined strategy evolution with topological evolution. They used asynchronous update rule both for evolution of strategy and structure by the Fermi function. In their model, topological evolution is manipulated in the following way: a pair of C–D or D–D are chosen at random, the one may compete with the other to rewire the link, rewiring being attempted to a random neighbor’s neighbor with certain probability determined by payoff difference between them. Whereas in our model, we argue that individuals are exclusively based on their self-interest. Even if the individual is a cooperator, it could not bear the exploitation by defectors. Furthermore, in our situation, individuals have enough inspection over their opponents because they are engaged in a $n$-round PDG. Subsequently, each agent can punish the most defective neighbor by dismissing the link, meanwhile seeks to establish links with cooperators. Especially, in our model the agents are endowed with limited memory abilities by which they can punish the most defective one. In addition, the way of associated time scales with respect to evolution of strategy and structure is different in our model from the previous related works. As aforementioned, after playing $n$-round PDG with neighbors, individuals update their local interactions according to the game results. Such timely feedback mechanism is ubiquitous in natural world. Besides, the adaption of network structure is much slower than evolution of strategies in our model. Such feature reflects the fact that individuals may not respond rapidly to the surrounding since maintaining and rewiring interactions are costly to them. In previous investigations, the time scale often is implemented in a stochastic manner. Although the implementation of time scales in the literature is equivalent to our model in a way, our method may be more plausible. Therefore, our model is different from the previous ones in these respects and captures the characteristics in real situation. In what follows, we investigate under which conditions cooperation may thrive by extensive numerical simulations. And also, we show the effects of the changing parameters in our model on the evolution of cooperation.

3. Simulation results and discussions

We consider $N$ individuals occupying the network vertices. Each interaction between two agents is represented by an undirected edge (a total of $N_E$). The social networks are evolving in time as individuals adapt their ties. The average connectivity $\langle k \rangle = 2N_E/N$ is conserved during the topological evolution since we do not introduce or destroy links. This point assumes a constrained resource environment, resulting in limited possibilities of network configurations. Besides, we impose that nodes linked by a single edge cannot lose this connection, thus the evolving networks are connected at all times. We calculated the amount of heterogeneity of the networks as $h = N^{-1} \sum_k k^2 N(k) - (\langle k \rangle)^2$ (the variance of the network degree sequences), where $N(k)$ gives the number of vertices with $k$ edges. Additionally, in order to investigate the degree–degree correlation pattern about the emerging social networks, we adopted the assortativity coefficient $r$ suggested by Newman [29],

$$
 r = \frac{M^{-1} \sum_i j_i k_i - [M^{-1} \sum_i \frac{1}{2} (j_i + k_i)]^2}{M^{-1} \sum_i \frac{1}{2} (j_i^2 + k_i^2) - [M^{-1} \sum_i \frac{1}{2} (j_i + k_i)]^2},
$$

here $j_i, k_i$ are the degrees of the vertices at the ends of the $i$th edge, with $i = 1, \ldots, N_E$. Networks with assortative mixing pattern, i.e., $r > 0$, are those in which nodes with large degree tend to be connected to other nodes with many connections and vice versa.

The interplay between network structure and game dynamics is implemented as following steps:

- **Step 1**: The evolution of strategy uses synchronous updating. Each agent plays the PDG with its immediate neighbors for consecutive $n$ rounds. After each round, each individual adapts its strategy according to Eq. (3), and records down the defection times of its every neighbors.

- **Step 2**: The update of individual’s local social interactions is asynchronous. $m$ agents are successively chosen at random to rewire the most adverse links (if any) as shown in Fig. 1.
Step 3: Repeat the above two steps until the population converges to an absorbing state (full cooperators or defectors), or stop repeating the above two steps after $10^5$ generations.

We start from a homogeneous random graph by using the method in Ref. [30], where all nodes have the same number of edges, randomly linked to arbitrary nodes. Initially, an equal percentage of cooperators and defectors is randomly distributed among the elements of the population. We run 100 independent simulations for the corresponding parameters $N$, $(k)$, $n$, $m$, $b$, and $\beta$. We also compute the fraction of runs that ended up with 100% cooperators. If the evolution has not reached an absorbing state after $10^5$ generations, we take the average fraction of cooperators in the population as the final result. Moreover, we observe the time evolution of the network structure and strategy, including the degree–degree correlation coefficient, the degree of heterogeneity, the frequency of cooperators, the fraction of C–C/C–D/D–D links, etc. Finally, we confirm that our results are valid for different population size $N$ and edge number $N_E$.

We report a typical time evolution of the network structure as a result of the adaption of social ties in Fig. 2, with relevant parameters $N = 10^4$, $(k) = 8$, $b = 1.2$, $n = 6$, $\beta = 50$, and $m = 100$. The emerging social network shows disassortative mixing pattern, indicating that large-degree nodes tend to be connected to low-degree nodes. The degree–degree correlation coefficient $r$ of the network we started from is zero. Once the network

![Fig. 2. Time evolution of the network structure as a result of the adaption of social ties: (a) the degree correlation coefficient, (b) the amount of heterogeneity, measured in terms of the variance of the degree distribution, (c) the frequency of cooperators, and (d) the fraction of C–C/C–D/D–D edges. The inset in panel (b) shows the cumulative degree distribution in steady state. The network evolution starts from a homogeneous random graph, in which all nodes have the same number of edges $(k)$, randomly linked to arbitrary nodes. The corresponding parameters are $N = 10^4$, $(k) = 8$, $b = 1.2$, $n = 6$, $\beta = 50$, and $m = 100.$]
structure adaption is in effect, disassortative mixing pattern will be developed. Since the rewiring process is attempted to a random neighbor’s neighbor, thus the nodes with large connectivity are more possible to be attached by others. Due to such “rich gets richer”, inhomogeneity is induced as shown in Fig. 2(b). The amount of the heterogeneity (degree variance) \( h \) increases in virtue of the rewiring process. The inset in Fig. 2(b) plots the cumulative degree distribution of the stationary (final) network, which exhibits high heterogeneity with a power-law tail. Fig. 2(c) displays the evolution of cooperation. We find that the frequency of cooperators decreases at first due to the temptation to defect, and then because of the adaptive interactions, the cooperation level thrives gradually and the population converges into an absorbing state of 100% cooperators. The viability of cooperation is also in part promoted by the heterogeneity of the underlying heterogeneity. From Fig. 2(d), we can see that local assortative interactions between cooperators is enhanced by structural updating, while assortative interactions between defectors and defectors is inhibited remarkably. The disassortativity between cooperators and defectors is promoted in the beginning by strategy updating, however, being diminished eventually by structural updating. Clearly, It is thus indicated that the interplay between strategy and structure will facilitate the emergence of cooperation.

Let us consider the effect of the amount of temptation to defect \( b \) to the evolution of cooperation. The relevant result is presented in Fig. 3. With increasing \( b \), the structural updating event must be sufficiently frequent to guarantee the survival of cooperators. In other words, the fraction of individuals chosen for updating social ties should be accordingly increased to ensure the sustainability of cooperators. When the value of \( b \) is enlarged, the defectors become more favorable by the nature selection. Nevertheless, with the aid of structural updating, a small fraction of surviving cooperators promotes them into hubs (large degree nodes), since they are attractive to neighborhood. Such co-evolution of strategy and structure leads to highly heterogeneous networks in which the cooperators become evolutionarily competitive as demonstrated in Refs. [13–15]. For fixed \( b \), we observed a critical value \( m_c \) for \( m \), above which the cooperator will wipe out defectors. For fixed number of rounds \( n \), the critical value \( m_c \) monotonously increases with increasing \( b \), as shown at the inset of Fig. 3. Therefore the prompt network adaption prevents cooperators from becoming extinct, further, resulting in an underlying heterogeneous social network which is the “green house” for cooperators to prevail under strategy dynamics. Consequently, the entangled co-evolution of strategy and structure promotes the evolution of cooperation among selfish individuals.

![Fig. 3. Fraction of cooperators at end as a function of \( m \) for different values of \( b \). We ran 100 simulations, starting from 50% cooperators. The values plotted correspond to the fraction of runs which ended with 100% cooperators. From left to right, \( b = 1.2, 1.6, 1.9 \), respectively. The inset plots the critical value \( m_c \) vs \( b \). \( N = 10^3 \), \( k = 4 \), \( n = 6 \), and \( \beta = 0.01 \).](image-url)
Furthermore, we investigated the effect of number of rounds $n$ to the emergence of cooperation. Fix fixed $m$ and other parameters, there exists a critical value for $n$, above which the cooperators will vanish as shown in Fig. 4. Indeed, although the structural updating promotes the cooperators to a certain extent, its role will be suppressed by the long-time strategy dynamics (corresponding to large $n$). In our case, strategy dynamics is synchronous while structural updating is asynchronous, namely, for each repetition in the simulations, strategy updating happens at a frequency of $N \cdot n$ while the structural updating occurs at a frequency of $m$. Hence the evolution of strategy is much more frequent than that of structure. Thus, with large $n$, able defectors outperform cooperators through strategy dynamics, even though the heterogeneity, resulting from structural updating, is positive to evolution of cooperation. This result illustrates that even if the evolution of network topology is less frequent than the evolution of strategy, cooperators still have chances to beat defectors under appropriate conditions.

As is well known, cooperation is promoted in the situation where individuals are constrained to interact with few others along the edges of networks with low average connectivity $[5,14,24]$. To understand the cooperation in real-world interaction networks of which the average connectivity is normally relatively high, one needs new insight into the underlying mechanism promoting cooperation. Here, the role of average connectivity to evolution of cooperation is inspected. In Fig. 5, it is shown that for increasing $\langle k \rangle$, the individuals must be able to promptly adjust their social ties for cooperation to thrive, corresponding to increasing $m$. Thus in order to explain the cooperation in communities with a high average number of social ties, the entangled co-evolution of network structure and strategy should be taken into account simultaneously. On static networks, maximum cooperation level occurs at intermediate average degree $[31]$. Moreover, when the connections are dense among individuals (large average connectivity $\langle k \rangle$), cooperators die out due to mean-field behavior. Conversely, our results suggest that even in highly-connected network, on account of the proposed structural adaption, cooperators can beat back the defectors and dominate the populations.

Finally, we report the influence of changing intensity of selection $\beta$ on the evolution of cooperation in Fig. 6. It is indicated that reducing $\beta$ will demote the influence of the game dynamics, thereby increase the survivability of the less fit. Clearly, the smaller the value of $\beta$ is, the smaller the critical value of $m$. In fact, for small $m$, cooperators' survival probability increases with decreasing $\beta$ although cooperators are generally less fit. Such increased survivability enhances assortative interactions between cooperators through network
structure adaption. As a result, the critical value of $m$ above which cooperators dominate defectors decreases with decreasing $b$.

4. Conclusions

In summary, we have studied the coupled dynamics of strategy evolution and the underlying network structure adaption. We provided a computational model in which individuals are endowed with limited
cognitive abilities in the \( n \)-round PDG—limited memories for recording the defection times of opponents. After the \( n \)-round game, \( m \) randomly chosen individuals are allowed to adjust their social ties based on the game results. The values of \( n \) and \( m \) are corresponding to the associated time scales of strategy dynamics and structural updating, respectively. We found that for a given average connectivity of the population and the number of rounds, there is a critical value for the fraction of individuals adapting their social interactions above which cooperators wipe out defectors. In addition, the critical value of \( m \) above which cooperators dominate defectors decreases with decreasing intensity of selection \( \beta \). Moreover, for increasing average connectivity, the individuals must be able to swiftly adjust their social ties for cooperators to thrive. Finally, the emerging social networks at steady states exhibit nontrivial heterogeneity which is the catalyst for emergence of cooperation among selfish agents. To a certain extent, our results shed some light on the underlying mechanism promoting cooperation among selfish individuals, and also provide an alternative insight into the properties accruing to those networked systems and organizations in natural world.

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