An online method for detection and reduction of chattering alarms due to oscillation

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Chattering alarms, which repeatedly and rapidly make transitions between alarm and normal states in a short time period, are the most common form of nuisance alarms that severely degrade the performance of alarm systems for industrial plants. One reason for chattering alarms is the presence of oscillation in process signals. The paper proposes an online method to promptly detect the chattering alarms due to oscillation and to effectively reduce the number of chattering alarms. In particular, a revised chattering index is proposed to quantify the level of chattering alarms; the discrete cosine transform-based method is used to detect the presence of oscillation; two mechanisms by adjusting the alarm tripping point and using a delay timer are exploited to reduce the number of chattering alarms. An industrial case study is provided to illustrate the effectiveness of the proposed method.

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1. Introduction

Alarm systems have been recognized as critical assets for safety and efficient operation of industrial plants, including power stations, oil refineries, and petrochemical facilities (Bransby & Jenkinson, 1998; Rothenberg, 2009). However, according to industrial surveys, e.g., those in Bransby and Jenkinson (1998) and Rothenberg (2009), operators of industrial plants often receive many more alarms than they can handle promptly, among which many belong to the nuisance alarms.

Chattering alarms are the most common forms of nuisance alarms. Rothenberg (2009) defines chattering alarms as the ones that activate ten or more times within 1 min. The ISA 18.2 standard (ISA, 2009) regards alarms that repeat more than three or more times in 1 min as chattering alarms. Two closely related nuisance alarms are the repeating alarms and fleeting alarms (Bransby & Jenkinson, 1998; EEMUA, 2007) that do not activate and clear as fast as chattering alarms, e.g., the repeating alarms activate ten or more times within 15 min (Rothenberg, 2009). It is a well-known fact that different types of process variables may have very different time scales in terms of variation dynamics, e.g., the Nyquist sampling period of a flow variable is usually in seconds, while that of a temperature variable may be in minutes. As a result, there is not a unanimous definition for chattering alarms in the literature. In this context, we refer chattering alarms, as well as repeating alarms and fleeting alarms, as the ones that repeatedly and rapidly make transitions between alarm and normal states in a short time period, where the time period is subject to the type of process variables in consideration.

Chattering alarms, as well as repeating alarms and fleeting alarms, have received an increasing attention from both industrial and academic communities. Burnell and Dicken (1997) introduced a detection and auto-shelving facility and a method of changing the alarm display list to handle repeating alarms. Bransby and Jenkinson (1998) (Appendix 10) and EEMUA (2007) (Appendix 9) discussed a number of mechanisms for dealing with repeating and fleeting alarms, including filtering, deadband, delay timer, shelving, etc. Hugo (2009) exploited the time series modeling technique to obtain adaptive alarm deadbands to reduce the number of chattering alarms. Kondaveeti, Izadi, Shah, Shook, and Kadali (2010) proposed a chattering index to quantify the degree of chattering alarms. Naghoosi, Izadi, and Chen (2011) developed a method to estimate the chattering index based on statistical properties of process variables as well as alarm parameters for the deadband or delay timer in use.

There are several reasons for the appearance of chattering alarms, such as the noise on a process variable that is operating close to an alarm tripping point. In this context, we focus on the reason of oscillation presented in process variables. Oscillation is one of phenomena frequently observed in process industries (Thornhill & Horch, 2007). If a signal is periodic or contains periodic components...
with well-defined amplitude and period, such as a sinusoidal wave contaminated by measurement noise, it is called an oscillatory signal (Choudhury, Shah, & Thornhill, 2008). If the oscillation amplitude is large enough to cause repeated crossings of the process variable over the alarm tripping point, then chattering alarms appear; see an industrial case study provided later in Section 7 for an illustration. Owing to the regularities of oscillation on the amplitude and period, it is expected that the chattering alarms caused by oscillation can be effectively reduced, which is the objective of this paper. To achieve this objective, a critical step is to detect the presence of oscillation.

The existing methods for detection of oscillation in a univariate signal include the integrated absolute error-based method (Hagglund, 1995; Thornhill & Hagglund, 1997), the correlation function-based method and the spectral peak-based method (Karra & Karim, 2009; Thornhill, Huang, & Zhang, 2003), the wavelet-based method (Matsuo, Sasaoka, & Yamashita, 2003), the autoregressive and moving-average model-based method (Salsbury & Singhal, 2005), the discrete cosine transform (DCT)-based method (Li, Wang, Huang, & Lu, 2010), the empirical mode decomposition (EMD)-based method (Srinivasana, Rengaswamy, & Miller, 2007) and its improved version (Srinivasana & Rengaswamy, 2012). Among these methods, the DCT-based method (Li et al., 2010) and the EMD-based method (Srinivasana & Rengaswamy, 2012) are perhaps the most advanced ones. In this paper, we would exploit the DCT-based method in Li et al. (2010) to detect the presence of oscillation. To make the paper self-sustained, the steps of the DCT-based method (with some modifications) are listed in Appendix A. Note that other oscillation detection methods, such as the EMD-based method, are applicable too (to be clarified later at Section 6).

The contribution of this paper is to propose an online method to promptly detect the presence of chattering alarms due to oscillation and to effectively reduce the number of these chattering alarms. In particular, a revised chattering index is provided to quantify the level of chattering alarms, with a correction of a drawback in the original chattering index proposed in Kondaveeti et al. (2010) and Naghoosi et al. (2011); two mechanisms, namely, the adjustment of alarm tripping and the usage of a delay timer, are exploited to reduce the number of chattering alarms due to oscillation that is detected via the DCT-based method. The reduction of chattering alarms certainly is accompanied by some costs. To control the costs within an acceptable level, the two mechanisms are designed with the consideration of requirements on three performance indices, namely, the false alarm rate (FAR), missed alarm rate (MAR) and averaged alarm delay (AAD). By contrast, the existing methods in Burnell and Dicken (1997), Bransby and Jenkinson (1998), EEMU A, 2007 and Hugo (2009) for handling chattering alarms are rather empirical, lack of quantitative measures on the benefits and costs.

The rest of the paper is organized as follows. Section 2 describes the considered problem. Section 3 discusses the mechanisms to reduce the number of chattering alarms. Section 4 investigates the run length distributions of chattering alarms. A revised chattering index is provided in Section 5. The proposed online method is presented in Section 6, and its effectiveness is illustrated via an industrial case study in Section 7. Finally, Section 8 provides some concluding remarks.

2. Problem description

Consider a discrete-time process signal \( x(t) \) with the sampling number \( t = 1, 2, \ldots \) and the sampling period \( h \) (a positive real number). Without loss of generality, \( x_{tp} \) is assumed to be a high-alarm tripping point associated with \( x(t) \). That is, an alarm signal \( x_{a}(t) \) takes the value ‘1’ if \( x(t) \) exceeds \( x_{tp} \), and the value ‘0’ if \( x(t) \) is smaller than \( x_{tp} \), i.e.,

\[
x_{a}(t) = \begin{cases} 
1, & \text{if } x(t) \geq x_{tp} \\
0, & \text{otherwise} 
\end{cases}
\] (1)

Chattering alarms may be arisen due to the presence of oscillation when \( x(t) \) is under the normal condition, as shown in Fig. 1. The oscillation causes many repeated crossings of \( x(t) \) over the tripping point \( x_{tp} \), so that chattering alarms appear as the false alarms, which severely degrade the performance of the alarm system on \( x(t) \). The performance of an alarm system can be measured by different indices. The basic indices are the number of alarms per hour, peak number of alarms per hour, number of high/low priority alarms per hour, and alarm acknowledge ratio (Bransby & Jenkinson, 1998; Rothenberg, 2009). For a univariate alarm system, its performance can be assessed by three indices, namely, the FAR, MAR and AAD (Xu, Wang, Izadi, & Chen, 2012). The FAR (MAR) is the probability of false (missed) alarms when \( x(t) \) is under the normal (abnormal) condition. The FAR and MAR measure the accuracy of an alarm system in detecting the normal and abnormal conditions. The AAD is the expected value of delay across the alarm activating time and the time instant that \( x(t) \) switches from the normal condition to abnormal. Thus, the AAD measures the alarm latency of an alarm system. A proper design of the alarm system, such as the selection of the alarm tripping point \( x_{tp} \) in (1), should satisfy certain requirements on the FAR, MAR and AAD, e.g., FAR \( \leq 5 \% \), MAR \( \leq 5 \% \) and AAD \( \leq 10 \) s.

Our objective is to design an online method that can promptly detect the presence of the chattering alarms due to oscillation and effectively reduce the number of these chattering alarms, while the requirements on FAR, MAR and AAD are satisfied.

3. Mechanisms to remove chattering alarms

Due to the regularity of oscillation, a proper mechanism can be taken to reduce the number of chattering alarms. Three mechanisms that are commonly adopted in practice are investigated here, namely, the shelving operation, adjustment of the alarm tripping \( x_{tp} \) and delay timer.

Let the probability distribution functions (PDFs) of the process signal \( x(t) \) in the normal and abnormal conditions be denoted by \( q(x) \) and \( p(x) \), respectively, as shown by the solid and dotted curves in Fig. 2. If \( x(t) \) is assumed to be independent and identically
distributed (IID), it is straightforward to obtain the FAR, MAR and AAD,
\[ \text{FAR} = q_1 := \int_{x_{tp}}^{+\infty} q(x) \, dx, \]  
\[ \text{MAR} = p_2 := \int_{-\infty}^{x_{tp}} p(x) \, dx, \]  
\[ \text{AAD} = h \cdot \frac{p_2}{1 - p_2}. \] (4)

There exist some tradeoffs among the FAR, MAR and AAD. The value of \( x_{tp} \) can be designed in an optimal manner based on the information of \( q(x) \) and \( p(x) \) as well as the requirements on the FAR, MAR and AAD. Xu et al. (2012) suggested the following approach,
\[ x_{tp} = \arg \min_{x_{tp}} J(x_{tp}) \]  
where
\[ J(x_{tp}) = \omega_1 \cdot \text{FAR} + \omega_2 \cdot \text{MAR} + \omega_3 \cdot \text{AAD}, \] (6)
subject to the constraints,
\[ \text{FAR} \leq \text{RFAR}, \ \text{MAR} \leq \text{RMAR}, \ \text{AAD} \leq \text{RAAD}. \] (7)

Here RFAR, RMAR and RAAD are the acceptable upper limits of FAR, MAR and AAD, respectively, and \( \omega_1, \omega_2 \) and \( \omega_3 \) are the weighting terms.

When \( x(t) \) stays at the normal condition and oscillation is present, the original PDF \( q(x) \) could be dramatically different from the one shown by the dash curve in Fig. 2. As a result, chattering alarms appear as the false alarms so that the FAR may increase severely. Then, the objective is to reduce the FAR subject to the tradeoffs among FAR, MAR and AAD.

The shelving operation is perhaps the mostly adopted one (Hollifield & Habibi, 2010). When oscillation is found to be present, the shelving operation temporarily isolates the chattering alarms into a shelf. In other words, there will be no alarms from \( x_a(t) \) to be presented to operators when \( x_a(t) \) is under the shelving operation. When oscillation is absent, \( x_a(t) \) is removed for the list of shelving operation. However, the shelving operation has a serious drawback. That is, when abnormal condition occurs during the time that \( x_a(t) \) is under the shelving operation, no alarms will be given, which is a potentially dangerous situation. In terms of the performance measures, this situation implies \( \text{MAR} = 100\% \) and \( \text{AAD} = +\infty \) so that the loss function \( J(x_{tp}) \) in (6) becomes unacceptable. Hence, Hollifield and Habibi (2010) (p. 166) stated that “it is essential that operators must know, each shift, which alarms have been removed from service and for how long”, which, however, increases the labor efforts of operators.

An alternative way is to adjust the alarm trip point \( x_{tp} \) temporarily to another value \( x_{tp} \) larger than the oscillation amplitude in order to accommodate the presence of oscillation. Doing so can decrease the FAR by removing the chattering alarms due to oscillation in \( x(t) \), but with the cost of increasing MAR and AAD, as implied by (4). If the adjustment of \( x_{tp} \) leads to a smaller loss function \( J(x_{tp}) \) than \( J(x_{tp}) \), then the adjustment of \( x_{tp} \) can be exploited, without the danger of using the shelving operation that may miss the detection of abnormal conditions.

The third choice is the \( m \)-sample delay timer. That is, an alarm will be raised/cleared if and only if more than \( m \) consecutive samples of \( x(t) \) are larger/smaller than the trip point \( x_{tp} \). For IID process signal \( x(t) \), the FAR, MAR and AAD for the \( m \)-sample delay timer are (Xu et al., 2012)
\[ \text{FAR} = \frac{q_1^{m-1} (1 - q_2^m)}{q_1^{m-1} (1 - q_2^m) + q_2^{m-1} (1 - q_1^m)}, \] (8)
\[ \text{MAR} = \frac{p_2^{m-1} (1 - p_1^m)}{p_2^{m-1} (1 - p_1^m) + p_1^{m-1} (1 - p_2^m)}, \] (9)
\[ \text{AAD} = h \frac{(1 - p_1^m - p_2 p_1^m)}{p_2 p_1^m}. \] (10)

where \( q_1 \) and \( p_2 \) are defined in (2) and (3), respectively, and \( p_1 := 1 - p_2 \) and \( q_2 := 1 - q_1 \). When oscillation is present, the factor \( m \) can be tuned to be larger than the half of oscillation period, so that no alarms will be triggered. Using the delay timer can reduce the FAR and MAR, but with an increment of the AAD, as implied by (8)–(10). Analogously to \( J(x_{tp}) \) in (6), another loss function \( J(m) \) can be defined. Thus, the usage of the delay timer may lead to a smaller loss function \( J(m) \) than the original one \( J(m = 1) \). In this case, the delay timer does not suffer from the drawback of the shelving operation too.

Based on the above observations and analysis, the adjustment of the alarm trip point and the delay timer are exploited further to reduce the number of chattering alarms, while the shelving operation is no longer considered in the sequel.

4. Run length distribution for alarm signals

This section introduces the run lengths of alarm signals that are the information source for the detection of chattering alarms.

The run length is defined for alarm signals that are generated in a way different from that in (1). That is, the alarm signal \( x_a(t) \) takes the value of ‘1’ only at the time instant when \( x(t) \) goes into the alarm state from the non-alarm state, i.e., for the high-alarm trip point \( x_{tp} \).
\[ x_a(t) = \begin{cases} 1, & \text{if } x(t-1) < x_{tp} \text{ and } x(t) \geq x_{tp} \\ 0, & \text{otherwise} \end{cases}. \] (11)

The run length, denoted as \( r \), is defined as the number of samples between two consecutive ‘1’s in \( x_a(t) \), i.e.,
\[ r := t_2 - t_1 \]

where
\[ x_0(t_1) = 1, \quad x_0(t_2) = 1, \quad \sum_{t=t_1}^{t_2} x_0(t) = 2, \quad \text{for} \ t_2 > t_1. \]

It is useful to investigate the distribution of the run length \( r \) for two special cases: (a) the process signal \( x(t) \) is IID and (b) \( x(t) \) is oscillatory.

Case-(a). If \( x(t) \) is IID, the total number of \( 1 \)'s in \( x_0(t) \) generated by (11) is a random variable denoted by \( X_0 \) with binomial distribution, i.e.,
\[
B(X_0; N, q_1(1-q_1)) = \frac{N!}{(N-X_0)!X_0!}(q_1(1-q_1))^{X_0}(1-q_1(1-q_1))^{N-X_0},
\]
where \( q_1 \) is defined in (2), and \( N \) is the length of collected samples of \( x_0(t) \). Owing to the triangular inequality, \( q_1(1-q_1) \) is less than \( 1/4 \), i.e.,
\[
q_1(1-q_1) \leq \left( \frac{q_1 + 1 - q_1}{2} \right)^2 = \frac{1}{4}. \tag{13}
\]

If the data length \( N \) is large and the probability of \( x_0(t) \) taking \( 1 \)'s is small (as implied by (13)), then the binomial random variable \( X_0 \) can be well approximated by Poisson distribution (Devore, 2005),
\[
P(X_0; N, \lambda) = \frac{e^{-\lambda}(\lambda^{X_0}/X_0!)}. \tag{14}
\]

Here \( \lambda = q_1(1-q_1) \) may be interpreted as the mean number of \( 1 \)'s per sample. Thus, the run length \( r \), as the time difference between consecutive \( 1 \)'s, has the exponential distribution (Devore, 2005),
\[
\mathbb{E}(r; \lambda) = \begin{cases} \frac{1}{\beta} e^{-\beta r}, & r > 0, \\ 0, & \text{elsewhere} \end{cases}
\]
where \( \beta = 1/\lambda = 1/(q_1(1-q_1)). \)

Case-(b). If \( x(t) \) is oscillatory, the oscillation period \( P \) (a positive integer) and amplitude \( M \) (a positive real number) can be defined as
\[
\mathbb{E} \{ x(t+kP) \} = \mathbb{E} \{ x(t) \}, \quad \forall k, \forall t,
\]
\[
M := \max_{t} \{ \mathbb{E} \{ x(t) \} \} - \min_{t} \{ \mathbb{E} \{ x(t) \} \},
\]
where \( \mathbb{E} \{ \cdot \} \) is the quasi-expectation operator to unify the stochastic and deterministic signals in the same roof,
\[
\mathbb{E} \{ x(t) \} = \lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} \mathbb{E} \{ x(t) \}.
\]

For a high-alarm tripping point \( x_{tp} \), it is reasonable to assume that \( x_{tp} \) is larger than the average level of \( x(t) \), denoted by \( m_0 \), but smaller than \( m_x + M/2. \) If \( x(t) \) is a deterministic periodic sequence, then the run length sequence is equal to a constant, the same as the oscillation period of \( x(t) \), i.e., \( r(t) = P, \forall t \). In practice, \( x(t) \) is usually contaminated by measurement noise, so that \( x_0(t) \) may take the value \( 1 \) when \( x(t) \) is close to \( x_{tp} \). As a result, the distances between consecutive points crossing \( x_{tp} \) may take two values of \( P_1 \) and \( P_2 \) for \( P_1 \leq P_2 \) and \( P_1 + P_2 = P \). Thus, the run lengths could take the values around \( P, P_1 \) and \( P_2 \) as well as some much smaller values due to the noise effects, which is illustrated in the next example.

**Example 1.** Let the process signal be generated as
\[
x(t) = \sin(2\pi ft) + e(t),
\]
where \( e(t) \sim \mathcal{N}(0, \sigma_e) \) is Gaussian white noise with zero mean and standard deviation \( \sigma_e \), and \( f \) is a constant equal to 0.01. The alarm tripping is \( x_{tp} = 0.6 \). Fig. 3(a)--(d) presents the histograms of the run lengths \( r \) in (12) for four values of \( \sigma_e = \{ 0, 0.05, 0.1, 1 \} \) with the data length \( N = 10,000 \). When \( \sigma_e = 0 \), all the run lengths are equal to the period \( P = 1/f = 100 \), as shown in Fig. 3(a). When \( e(t) \) is present, the points of \( \sin(2\pi ft) \) close to \( x_{tp} \) have certain probabilities to trigger an alarm due to the noise effect; as an illustration, some parts of \( x(t) \) and its alarm signal \( x_0(t) \) for \( \sigma_e = 0.05 \) are shown in Fig. 4. Thus, the histogram of the run length may consist of four parts as shown in Fig. 3(b) and (c). When \( \sigma_e \) is increasing, \( e(t) \) may overwhelm the oscillatory component so that \( x(t) \) is dominated by the Gaussian white noise \( e(t) \). As expected, the distribution of the run length approaches to the exponential distribution in (14), as shown in Fig. 3(d) for \( \sigma_e = 1 \). The observation on the run length distribution for oscillatory signals in this example will be used later at Step 3 of the proposed method in Section 6 where the data segment is determined for the detection of oscillation.

The run length \( r \) in (12) is exploited in the next section to formulate an index to detect the occurrence of chattering alarms.
5. Revised chattering index

This section proposes a revised chattering index. For the time being, let the sampling period $h$ of $x(t)$ be 1 s.

A chattering index was proposed in Kondaveeti et al. (2010) and Naghoosi et al. (2011),

$$\psi = \frac{\sum AC_r}{r},$$  \hspace{1cm} (15)

where $r$ is the run length in (12) and $AC_r$ is the total number of $r$. The index $\psi$ may lead to an exaggerated impression on the frequency of appearance of alarms. For instance, if there are only two alarms with the time distance 2 s between them in the time duration of 100 s, then $r=2$, $AC_r=1$ and $\psi=0.5$. However, if the alarms occur every 2 s in the 100 s, then the chattering index is also equal to 0.5. Both cases reach at the upper limit of $\psi$ as the range of $\psi$ is [0, 0.5] for the alarm generation mechanisms in (11). This drawback is clearly due to the absence of the number of samples in the calculation of $\psi$. Hence, we would like to revise the chattering index as

$$\eta = \frac{2\sum AC_r}{N},$$ \hspace{1cm} (16)

where $N$ is the data length of the alarm signal. The coefficient 2 is to make the range of $\eta$ to be [0, 1].

A cutoff threshold of $\psi$ is $\psi_0 = 3/60 = 0.05$ alarms/s (Kondaveeti et al., 2010; Naghoosi et al., 2011), based on a rule of thumb from ISA 18.2 standard that alarms that repeat more than three times per minute are considered chattering. This cutoff threshold does not consider the effect of the weighting $1/r$ that penalizes small run lengths. In terms of $\eta$ in (16), if the same rule of thumb is applied, the worst scenario is that the three alarms in one minute are positioned closest to each other, so that the chattering index is

$$\eta = \frac{6 \cdot \frac{1}{60}}{60} = 0.05.$$

If oscillation is present and the three alarms are spread evenly in 1 min, then the chattering index is

$$\eta = \frac{6 \cdot \frac{1}{60}}{60} = 0.005.$$ \hspace{1cm} (17)

Thus, $\eta = 0.005$ is taken as the cutoff threshold. That is, if the calculated chattering index for a given data segment of $x_A(t)$ is larger than 0.005, then we claim that the segment contains chattering alarms, and some proper actions need to be taken to deal with the chattering alarms.

**Example 2.** Four sets of alarm signals with the sampling period $h=1$ sec are presented in Fig. 5. The total numbers of alarms, denoted by $X_A$ and the chattering indices are listed in Table 1.

For the alarm signals in Fig. 5(a) and (b), both have 2 alarms; however, the alarms in Fig. 5(b) occur quite close to each other; as a result, the chattering index $\eta$ in (16) for $x_A(t)$ in Fig. 5(b) is higher than that in Fig. 5(a). The original chattering index $\psi$ in (15) for Fig. 5(b) is quite large, $\psi = 0.3333$, leading to an incorrect conclusion that chattering alarms present.

For the alarm signal in Fig. 5(c), the alarm frequency is $9/500 = 0.0180$, and $\eta = 0.0024$ is smaller than the cutoff threshold 0.005. For the alarm signal in Fig. 5(d), the alarm frequency is $11/185 = 0.0595$, and the alarms are distributed quite evenly; thus, $\eta$ for Fig. 5(d) is close to the cutoff threshold 0.005. The chattering index $\eta$ says that chattering alarms are present in Fig. 5(d), and absent in Fig. 5(c), while the original chattering index $\psi$ incorrectly concludes that chattering alarms are present in both Fig. 5(c) and (d), and $\psi$ for Fig. 5(c) is even larger than that for Fig. 5(d).

<table>
<thead>
<tr>
<th>$X_A(t)$</th>
<th>$N$</th>
<th>$X_A$</th>
<th>$\eta$</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 5(a)</td>
<td>207</td>
<td>2</td>
<td>7.8552 × 10^{-5}</td>
<td>0.0081</td>
</tr>
<tr>
<td>Fig. 5(b)</td>
<td>234</td>
<td>2</td>
<td>0.0028</td>
<td>0.3333</td>
</tr>
<tr>
<td>Fig. 5(c)</td>
<td>500</td>
<td>9</td>
<td>0.0024</td>
<td>0.0754</td>
</tr>
<tr>
<td>Fig. 5(d)</td>
<td>185</td>
<td>11</td>
<td>0.0064</td>
<td>0.0596</td>
</tr>
</tbody>
</table>

![Fig. 5. Four alarm signals $x_A(t)$ in Example 2 with the sampling period $h = 1$ s.](image)
It is straightforward to see from the above results that the original chattering index \( \psi \) in (15) does not take the number of samples into consideration, and thus may lead to incorrect conclusions, while the chattering index \( \eta \) in (16) can measure the actual level of chattering alarms. 

**Remark 1.** In the above discussion, the sampling period is assumed to be one second. As stated in Section 1, different types of process variables may have different time scales in terms of variation dynamics. As a result, the sampling period for some process variables could be in minutes or hours. In addition, there are several common causes for oscillation, including sticky control valves, oscillatory external disturbances, loop interaction and aggressive control (Karra & Karim, 2009; Thornhill & Horch, 2007); thus, the oscillation periods may vary greatly in practice. Taking some industrial control loops in an international database\(^1\) accompanied with the book (Jelali & Huang, 2010) for examples, a level control loop (cdata.chemicals.loop4 in the database) is oscillatory with a period of about 16 s due to a controller tuning problem, and a flow control loop (cdata.chemicals.loop5 in the database) is oscillatory with a period of about 10 s due to a sticky control valve. As another example, in the industrial case study in Section 7, the drum level is oscillatory with a period of about 18 min due to the loop interaction. Therefore, it is reasonable not to confine the sampling period to the time scale of seconds or minutes. Since the chattering index \( \eta \) in (16) is invariant to the sampling period, \( \eta \) is equally applicable in general. The cutoff threshold 0.005 for the chattering index \( \eta \) could still be a reasonable choice, by generalizing the rule of thumb from the ISA 18.2 standard as “for a process variable with the sampling period \( h \), alarms that repeat more than three times in the duration of 60\( h \) are considered as chattering alarms”.

6. The proposed method

This section proposes an online method to detect the presence of chattering alarms due to oscillation and to reduce the number of these chattering alarms.

The method has the option of using two mechanisms to deal with chattering alarms, namely, adjusting the alarm tripping point or using an \( m \)-sample delay timer. If the adjustment of \( xtp \) is adopted, there are two sets of alarm signals \( x_a(t) \) and \( x_a(t)^{\text{m}} \) resulting from (11) by using the original tripping point \( xtp \) and the adjusted one \( xtp \), respectively. If the \( m \)-sample delay timer is used, then there are also two sets of alarm signals \( x_a(t) \) from (11) and \( x_a(t)^{\text{m}} \) that is generated analogously to (11) as

\[
x_a^{(m)}(t) = \begin{cases} 
1, & \text{if } x(t - m) < xtp \text{ and } x(t - m + 1 : t) \geq xtp \\
0, & \text{otherwise}
\end{cases}
\]

Here \( x(t - m + 1 : t) \) is a short notation for the sample set \( \{x(t - m + 1), x(t - m + 2), \ldots, x(t)\} \). It is expected that the two mechanisms can effectively reduce the number of chattering alarms, so that only \( x_a^{(p)}(t) \) or \( x_a^{(m)}(t) \), instead of \( x_a(t) \), is presented to operators.

We make the following assumptions:

A1. The normal and abnormal conditions of \( x(t) \) are associated with different means.

A2. The past measurements of \( x(t) \) that can well represent the normal and abnormal conditions of \( x(t) \) are available.

A3. Oscillation appears for a certain time period when \( x(t) \) is under the normal condition, leading to the appearance of chattering alarms.

A4. \( x(t) \) is IID except for the case that \( x(t) \) is oscillatory.

A5. The values of RFAR, RMAR and RAAD as the upper limits of FAR, MAR and AAD are known a priori.

Assumption A1 says that the normal and abnormal conditions of \( x(t) \) are differentiated by a change on the mean value, which is commonly observed in practice. The past measurements of \( x(t) \) in Assumption A2 is used for estimating the PDFs of \( x(t) \) in the normal and abnormal conditions. Owing to Assumptions A3 and A4, the mechanism of adjusting the alarm tripping point \( xtp \) or using the \( m \)-sample delay timer can be designed to reduce the number of chattering alarms so that the FAR can be decreased; meanwhile, the requirements on the MAR and AAD can still be satisfied, because the expressions of the MAR and AAD in (9) and (10) are irrelevant to the normal PDF \( q(x) \) so that the MAR and AAD for the adjusted alarm tripping point or the \( m \)-sample delay timer can be calculated. Note that the FAR, MAR and AAD are defined for the alarm signal generated in (1), while the chattering index is based on that in (11).

The proposed method consists of the following steps:

Step 1. Estimate the normal and abnormal PDFs \( q(x) \) and \( p(x) \) based on the past measurements of \( x(t) \) as assumed in Assumption A2, and obtain the upper limits of \( xtp \) and \( m \) as \( xtp^{(n)} \) and \( m^n \) from (8) to (10) to satisfy the requirements on the FAR, MAR and AAD, i.e., \( xtp^{(n)} \) and \( m^n \) are the upper limits of \( xtp \) and \( m \) respectively to make the inequalities in (7) hold.

Step 2. Initialize the adjusted alarm tripping point \( xtp \) the same as the original one, i.e., \( x_{tp} = xtp \), or use \( m = 1 \) for the \( m \)-sample delay timer.

Step 3. Let the current sample index be denoted as \( n \). The current task is to determine the data segment of process signal \( x(n - N + 1 : n) \) and its alarm signal \( x_a(n - N + 1 : n) \) for the subsequent steps.

In order to have a reliable detection of oscillation in a data segment, it is recommended based on the period regularity test (A5) that there are at least 10 periods in the segment \( x(n - N + 1 : n) \). If \( x(n) \) is oscillatory, then there is one single ‘1’ appeared in each oscillation period. Thus, the value of \( N \) can be determined to make the data segment \( x_a(n - N + 1 : n) \) contain 10 alarms. However, due to the presence of noise, multiple ‘1’s can appear in an oscillation period. The value of \( N \) is determined as the smallest one to satisfy the equality

\[
\left\lfloor \sum_{t=n-N+1}^{n} x_a(t) \right\rfloor - R = 10.
\]

where \( R \) is the number of the run lengths that satisfy the inequality

\[
r(l) < \max \left( r_0, \frac{\max r(l)}{2} \right).
\]

Here \( r(l) \) is the sequence of the run length \( r \) in (12) for \( l = 1, 2, \ldots, L \). The inequality (19) is based on the observation of the distribution of run length \( r \) in Example 1. It mainly says that the run lengths being smaller than one half of the maximum of \( r(l) \) should be ignored. The parameter \( r_0 \) is a small number to avoid a possible degenerated case, e.g., \( r_0 = 5 \). That is, if, the minimum value of period for an oscillation of interest is equal to 2\( r_0 \), we can ignore these alarms occurring so frequently such that their run lengths are smaller than \( r_0 \). Alarms are sometimes absent for a long time; in order to avoid too large value of \( N \), we select an integer \( N^m \) as the upper bound of \( N \), i.e., \( N \leq N^m \).

Step 4. For the selected data segment \( x(n - N + 1 : n) \), obtain the two sets of alarm signals \( x_a(n) \) and \( x_a^{(p)}(n) \) if the adjustment of \( xtp \) is used, or the other two sets \( x_a(n) \) and \( x_a^{(m)}(n) \) if the \( m \)-sample delay timer is exploited. The chattering indices for \( x_a(t), x_a^{(p)}(n) \) and

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\(^1\) Available online at [http://www.ualberta.ca/ bhuang/book2.htm](http://www.ualberta.ca/bhuang/book2.htm)
\( x_a^{(m)}(n) \) are denoted as \( \eta, \eta^{(m)} \) and \( \eta^{(m)} \), respectively. Next, the alarm tripoint \( x_{tp}^{(m)} \) or the coefficient \( m \) in the \( m \)-sample delay timer is updated for different cases as follows:

- **Case I.** If \( \eta^{(p)} < \eta_0 \) (\( \eta^{(m)} < \eta_0 \)) and \( \eta \geq \eta_0 \), that is, using the adjusted tripoint \( x_{tp} \) (the \( m \)-sample delay timer) can effectively reduce the number of chattering alarms, then preserve the previous value of \( x_{tp} \) or \( m \). Here \( \eta_0 \) is the cutoff threshold, equal to 0.005 in (17).

- **Case II.** If \( \eta^{(p)} \geq \eta_0 \) (\( \eta^{(m)} \geq \eta_0 \)) and \( \eta \geq \eta_0 \), that is, there are too many chattering alarms even for the adjusted tripoint \( x_{tp} \) (the \( m \)-sample delay timer), then the DCT-based method presented in Appendix A is used to detect the presence of oscillation in \( x(n - N + 1 : n) \).

(a) If an oscillation is detected, then update \( x_{tp} \) or \( m \) as

\[
\hat{x}_{tp} = \min \left( x_{tp}^{(m)}, \ x_m + 0.5M + \gamma_M \right),
\]

\[
m = \min \left( m^{(m)}, \ \left[ 0.5P + \gamma_P \right] \right).
\]

(b) If no oscillation is detected, then the chattering alarms are not caused by oscillation, and there is no need to adjust the tripoint or use the delay timer, so that \( \hat{x}_{tp} \) or \( m \) returns to the original value, i.e., \( \hat{x}_{tp} = x_{tp} \) or \( m = 1 \).

- **Case III.** If \( \eta^{(p)} \leq \eta_0 \) (\( \eta^{(m)} \leq \eta_0 \)) and \( \eta < \eta_0 \), that is, there are few chattering alarms even for the original tripoint \( x_{tp} \), then we do not need to use the delay timer or adjust the tripoint, i.e., \( \hat{x}_{tp} = x_{tp} \) or \( m = 1 \).

---

**Step 5.** With a new sample \( x(n+1) \), repeat Steps 3 and 4.

**Remark 2.** Step 3 determines the data segment \( x(n - N + 1 : n) \), to which the DCT-based method is applied at Step 4 to detect the presence of oscillation. Thus, other off-line oscillation detection methods, capable of providing the estimates of oscillation amplitude and period, can be applied to the data segment \( x(n - N + 1 : n) \) too.

7. **Industrial case study**

An industrial case study is presented here to illustrate the effectiveness of the proposed method.

The measurements of an industrial process variable \( x(t) \) were collected with the sampling period \( h = 1 \) minute at a large-scale petro-chemical plant affiliated to Sinopec Yangzi Petro-chemical Co., China. The process variable \( x(t) \) is the level of a drum under a closed-loop control depicted in Fig. 6. Owing to the material and energy connections between the drum and tower, the two level control loops, denoted by controllers LTC1 and LTC2 in Fig. 6, are interactive to each other. As a result, oscillations with the same period of about 18 min (to be estimated later by the DCT-based method) often arise in the levels of the drum and tower (together with about twenty other process variables associated with the drum, tower, and heat exchanger), as shown by the collected data of the two control loops in Fig. 7. Once the oscillations appear, plant operators usually take the action of switching the control loop LTC2 into the manual mode to break the loop interaction, and hence the oscillations disappear gradually.

A high alarm is configured for the drum level \( x(t) \) with the alarm tripoint \( x_{tp} = 46.0 \). Due to the above-mentioned loop interaction, \( x(t) \) often exhibits oscillatory behaviors and leads to chattering alarms, as shown by some collected data of \( x(t) \) and its alarm signal \( x_a(t) \) in Fig. 8. The data can be downloaded for academic studies online.\(^2\)

The first step of the proposed method is to estimate the normal and abnormal PDFs of \( x(t) \) and to obtain the upper limits of \( x_{tp} \) and \( m \). Some typical data sets for \( x(t) \) in the normal and abnormal conditions are presented in Fig. 9. By exploiting the method of estimating PDFs in Xu et al. (2012) (Section 5 therein), the estimates of the normal and abnormal PDFs \( q(x) \) and \( p(x) \) are obtained as shown in Fig. 10, based on the data samples in Fig. 9. For the alarm tripoint \( x_{tp} = 46.0 \), the estimates of \( q_1 \) in (2) and \( p_2 \) in (3) are \( q_1 = 0.0083 \) and \( p_2 = 0.0015 \), respectively. Suppose that the requirements on the FAR, MAR and AAD are

\[
\text{FAR} < 5\%, \quad \text{MAR} < 10\%, \quad \text{AAD} < 30 \text{ min}.
\]

---

Under Assumptions A4 and A5, using the estimates of $q(x)$ and $p(x)$ in Fig. 10, the upper limits of $x_0$ and $m$ are obtained as $x_0^m = 47.5$ and $m^H = 30$ from (8) to (10) based on the requirements in (25).

Second, the proposed method is applied to the real-time measurements of $x(t)$ shown in Fig. 11. The obtained results are listed in Table 2. For a better visualization, the enlarged versions of $x(t)$ in Fig. 11 are provided in Figs. 12–15, for different data segments. In Table 2, $\eta$ and $X_0$, respectively are the chattering index and the total number of ‘1’ s in $x_0(t)$ for the original alarm configuration with $x_0 = 46.0$ and $m = 1$, while $\eta^{(p)}, X^{(p)}_0$ and $\eta^{(m)}, X^{(m)}_0$, respectively are the counterparts from the two mechanisms, namely, the adjustment of $x_0$ and the $m$-sample delay timer.

For the first two data segments $x(1:500)$ and $x(501:1000)$ shown in Fig. 12, the initialization is $x_0 = 46.0$ or $m = 1$, and there are no alarms or one single alarm; as a result, the lengths of the first two data segments reach at the upper bound $N_H = 500$. The chattering indices $\eta^{(p)}$ ($\eta^{(m)}$) and $\eta$ are smaller than the cutoff threshold $\eta_0 = 0.005$; Case III is applicable so that we stay with $x_0 = 46.0$ ($m = 1$).

For the third data segment $x(1001:1399)$ shown in Fig. 13, there are 20 alarms and the chattering index $\eta^{(p)} = 0.0130$ ($\eta^{(m)} = 0.0130$) that is higher than the cutoff threshold $\eta_0 = 0.005$, leading to Case II. The DCT-based method does not detect the presence of oscillation in the third data segment, which is a correct result, because the oscillation therein is developing from lower amplitudes to larger ones and the oscillatory amplitudes do not have sufficient regularity (the inequality (A.9) for the amplitude regularity test does not hold). Thus, Case II-b is applicable, i.e., $x_0 = 46.0$ ($m = 1$).

For the fourth data segment $x(1400:1600)$ also shown in Fig. 13, there are 14 alarms and the chattering index $\eta^{(p)} = 0.0190$ ($\eta^{(m)} = 0.0190$) that is higher than $\eta_0 = 0.005$. The DCT-based method detects the presence of oscillation with amplitude $M = 2.8264$ and standard deviation $M_{std} = 0.5178$ around the average level $x_m = 44.9835$, and with period $P = 18.3810$ and standard deviation $P_{std} = 1.2032$. For RFAR = 5%, Eqs. (23) and (24) yield $\gamma_M = 1.1577$ and $\gamma_P = 2.6904$, respectively. Case II-a is applicable so that Eqs. (20)
Fig. 10. The estimated normal and abnormal PDFs \( q(x) \) (solid) and \( p(x) \) (dash).

Fig. 11. The process signal \( x(t) \) (solid) and its alarm trippoint \( x_p \) (dash) with the presence of oscillations (the sampling period \( h = 1 \) min).

**Table 2**

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<th>( x(t) )</th>
<th>( N )</th>
<th>( X )</th>
<th>( \eta \times 10^{-2} )</th>
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<th>( x_{\text{p}} )</th>
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Fig. 12. The process signal \( x(t) \) and alarm signal \( x_p(t) \) in the first and second segments (the sampling period \( h = 1 \) min).

Fig. 13. The process signal \( x(t) \) and alarm signal \( x_p(t) \) in the third and fourth segments (the sampling period \( h = 1 \) min).

Fig. 14. The process signal \( x(t) \) in the 5th–17th segments (the sampling period \( h = 1 \) min).
and (21) with $x_{tp}^{iH} = 47.5$ and $m^H = 30$ give $\bar{x}_{tp} = 47.5$ and $m = 12$, respectively. For the 5th–17th data segments shown in Fig. 14, the chattering index $\eta$ for $x_{tp} = 46.0$ is always larger than $\eta_0 = 0.005$, while $\eta^{(1)} (\eta^{(m)})$ for $\bar{x}_{tp} = 47.5 (m = 12)$ is smaller than $\eta_0 = 0.005$, so that Case I is applicable and the value of $\bar{x}_{tp} = 47.5 (m = 12)$ is preserved for these data segments.

For the 18th data segment shown in Fig. 15, both $\eta^{(1)} \leq 0.005$ and $\eta$ are smaller than the cutoff threshold $\eta_0 = 0.005$; Case III is applicable so that $\bar{x}_{tp}$ or $m$ returns to $\bar{x}_{tp} = x_{tp} = 46.0$ or $m = 1$ for the next data segment. Case III prevails for the rest data segments in Fig. 15.

The proposed method detects the presence of chattering alarms due to oscillation promptly at the four data segment, and removes most of these chattering alarms from the 5th to 17th data segments. In particular, the adjustment of $\bar{x}_{tp}$ (or the $m$-sample delay timer) reduces the total number of alarms in the 5th-17th data segments from 260 to 12 (or from 260 to 3). This is a significant improvement in terms of reduction the number of chattering alarms due to oscillation. Meanwhile, the quantitative requirements on MAR and AAD in (25) are satisfied, because $\bar{x}_{tp} = 47.5$ and $m = 1$ are not larger than their upper limits $x_{tp}^{iH} = 47.5$ and $m^H = 30$.

8. Conclusion

The paper proposed an online method to detect the presence of chattering alarms due to oscillation in process signals, and to reduce the number of these chattering alarms. The method measured the level of chattering alarms via a revised chattering index, exploited the DCT-based method to detect the presence of oscillation, and reduced the number of chattering alarms by adjusting the alarm triptipoint or using an $m$-sample delay timer. The effectiveness of the proposed method was illustrated via an industrial case study.

The proposed method could be further improved. First, it may be necessary to add a step to differentiate special types of oscillation for choosing a proper mechanism. For instance, some oscillatory signals are accompanied with slowly time-varying trends, for which using a delay timer is more suitable than adjusting the alarm triptipoint. Second, as discussed in Bransby and Jenkinson (1998) and EEMUA (2007), there are other techniques along with adjusting the alarm triptipoint and introducing an $m$-sample delay timer to handle chattering alarms. These techniques certainly deserve a careful investigation.

**Acknowledgment**

The authors would like to thank Sinopec Yangzi Petro-chemical Co., Nanjing, China, for providing industrial data used in this study.

**Appendix A. DCT-based method**

Given the data segment $x(n - N + 1 : n)$ determined at Step 3 of the proposed online method in Section 6, the DCT-based method consists of the following steps.

Step A1. Remove the average level $x_m$, i.e.,

$$x_d(1 : N) = x(n - N + 1 : n) - x_m$$

where

$$x_m = \frac{1}{N} \sum_{n = -N+1}^{n} x(t),$$

and perform the DCT on the segment $x_d(1 : N)$ to obtain the DCT component $y(k)$, i.e.,

$$y(k) = w(k) \sum_{t = 1}^{N} x_d(t) \cos \left(\frac{2\pi(t - 1)(k - 1)}{2N}\right), \quad k = 1, \ldots, N$$

where

$$w(k) = \begin{cases} 1 \sqrt{N}, & k = 1, \\ \sqrt{2 \frac{N}{k}}, & 2 \leq k \leq N. \end{cases}$$

Step A2. Computer a high sea-level (SL) parameter from $y(k)$,

$$SL = 3S_y,$$

where $S_y$ is the sample standard deviation of $y(k)$,

$$S_y = \sqrt{\frac{1}{N} \sum_{k = 1}^{N} \left( y(k) - \frac{1}{N} \sum_{k = 1}^{N} y(k) \right)^2 }.$$

Suppress the elements of $y(k)$ that are smaller than the high SL parameter in (A.2) to yield a sequence $y_f(k)$ as

$$y_f(k) = \begin{cases} y(k), & |y(k)| \geq SL, \\ 0, & |y(k)| < SL. \end{cases}$$

Generate the $i$-th DCT component $y_{if}(k)$ having the same length $N$ as $y(k)$ for $i = 1, 2, \ldots, I$,

$$y_{if}(k) = \begin{cases} y_{if}(k), & \text{for } k_s,i \leq k \leq k_e,i, \\ 0, & \text{otherwise}, \end{cases}$$

where $y_{if}(k)$ stands for the $i$-th segment of $y(k)$, and its the starting point $k_s,i$ and the ending point $k_e,i$ are determined by the conditions:

$$y_{if}(k) \neq 0 \text{ and } y_{if}(k_s,i - k_0) = 0, \text{ for } k_0 = 1,$$

$$y_{if}(k) \neq 0 \text{ and } y_{if}(k_e,i + k_0) = 0, \text{ for } k_0 = 1, 2, 3, 4,$$

$$k_s,i \leq k_e,i.$$
Step A3. Perform the inverse discrete cosine transform (IDCT) on 
\( y_i(t) \) to get the i-th component \( x_i(1 : N) \) as
\[
x_i(t) = \sum_{k=1}^{N} w(k) y_i(k) \cos \left( \frac{\pi(2t-1)(k-1)}{2N} \right), \quad t = 1, \ldots, N.
\] (A.4)

Find the zero crossing position sequence of \( x_i(1 : N) \), denoted as \( z(l) \) for \( l = 1, 2, \ldots, L \), i.e., \( z(l) = t \), where the sample index \( t \) satisfies the condition \( x_i(t)x_i(t+1) \leq 0 \). Obtain the period sequence \( T_I(l) \) as
\[
T_I(l) := 2(z(l+1) - z(l)).
\]

Check if \( x_i(1 : N) \) can pass the period regularity test
\[
R_{TI,\alpha} > 3
\] (A.5)

where
\[
R_{TI,\alpha} := \frac{\sqrt{\sum_{l=1}^{L-\alpha/2} (T_I(l) - \bar{T}_I)^2}}{\sqrt{\bar{T}_I}}.
\] (A.6)

Here \( \bar{T}_I \) and \( S_T \) respectively are the sample mean and standard deviation of \( T_I(l) \),
\[
\bar{T}_I = \frac{1}{L} \sum_{l=1}^{L} T_I(l), \quad S_T = \sqrt{\frac{1}{L-1} \sum_{l=1}^{L} (T_I(l) - \bar{T}_I)^2}.
\] (A.7)

and \( \sum_{l=1}^{L-\alpha/2} \) is the 100\( -\alpha \)th percentile of a chi-square distribution with \( L-1 \) degree of freedom. Here \( \alpha \) is a small positive real number, e.g., \( \alpha = 0.05 \). If none of \( x_i(1 : N) \) for \( i = 1, 2, \ldots, L \) passes the period regularity test (A.5), then \( x(n - N + 1 : n) \) is concluded to be non-oscillatory; otherwise, proceed to Step A4.

Step A4. Repeat Step A2 for a low SL parameter, \( SL = S_y \), find the high-SL DCT component \( y_i(k) \) in (A.3) that has passed the period regularity test (A.5), and extract the low-SL DCT component \( y_i(k) \) for \( j = 1, 2, \ldots, J \) that has the same maximum absolute value as \( y_i(k) \), i.e., \( y_{j,\max} = y_{i,\max} \), where \( y_{i,\max} := \max (|y_i(k)|) \), \( y_{j,\max} := \max (|y_j(k)|) \).

Step A5. Perform the IDCT on \( y_i(k) \) to get \( x_i(1 : N) \) as in (A.4), and conduct the period regularity test on \( x_i(1 : N) \) analogously to Step A3.

Step A6. Find the low-SL component \( x_{j,\max}(1 : N) \) having the largest fitness
\[
F(x, x_{j,\max}) = \max_j F(x, x_j)
\]
among these \( x_i(1 : N) \)'s that succeed in the period regularity test. Here the fitness of \( x_i(t) \) to \( x(t) \) is defined as
\[
F(x, x_j) = 100(1 - \frac{\|x(t) - x_j(t)\|_2}{\|x(t) - E[x(t)]\|_2})
\]
where \( \| \cdot \|_2 \) is the Euclidean norm. Locate the high-SL component \( x_{j,\max}(1 : N) \) associated with \( x_{j,\max}(1 : N) \) as well. Choose the oscillation period \( P \) with its standard deviation \( P_{\text{std}} \) as
\[
P = P_{\text{std}} \begin{cases} 
T_{j,\max} + S_{T_{j,\max}} & \text{if } R_{T_{j,\max},\alpha} \geq R_{T_{j,\max},\alpha} \\
T_{j,\max} - S_{T_{j,\max}} & \text{otherwise}
\end{cases}.
\] (A.8)
where \( R_{T_{j,\max},\alpha} \) and \( R_{T_{j,\max},\alpha} \) are the regularity indices (defined in (A.6)) for \( x_{j,\max}(1 : N) \) and \( x_{j,\max}(1 : N) \), respectively. The standard deviation

Step A7. Calculate the magnitude sequence \( M(l) \) as
\[
M(l) = \max \left( \{ x_i(t) \}_{i=1}^{L} \right) - \min \left( \{ x_i(t) \}_{i=1}^{L} \right)
\]

for \( l = 1, 2, \ldots, L \). Let the sample mean and standard deviation of \( M(l) \) be \( \bar{M} \) and \( S_M \), defined similarly as \( \bar{T}_I \) and \( S_T \) in (A.7). Analogously to (A.5), if the magnitude regularity test
\[
R_{M,\alpha} := \frac{\sqrt{\sum_{l=1}^{L-\alpha/2} (M(l) - \bar{M})^2}}{\sqrt{\bar{M}}} > 2.73
\] (A.9)
is passed, then \( x(n - N + 1 : n) \) is concluded to be oscillatory with the period \( P \) and standard deviation \( P_{\text{std}} \) in (A.8), and the amplitude \( \bar{M} \) and standard deviation \( S_M \) around the average level \( x_m \) in (A.1); otherwise, it is concluded that \( x(n - N + 1 : n) \) is not oscillatory.

References


