Performance Assessment of PID Control Loops based on IMC Tuning Rule

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Abstract: In industrial feedback control loops, setpoint changes are often in the form of step, ramp or other general types, and controllers are usually restricted to the PID form. This paper establishes the lower bounds of integrated absolute errors (IAEs), based on the widely-used internal model control (IMC) principle, from closed-loop responses subject to these setpoint changes. Taking the lower bound as a benchmark, an IMC-IAE-based index is proposed to assess the performance of PID controllers. Numerical and experimental examples, as well as an industrial case study, are provided to verify the lower bound as the performance benchmark, and to illustrate the effectiveness of using the proposed index for performance assessment of PID control loops.

Keywords: PID controller, IAE, IMC, performance assessment

1. INTRODUCTION

Many industrial feedback control loops suffer from performance problems, possibly due to incorrect tuning, large unmeasured disturbances, inappropriate control structures [5]. These problems may cause inferior products, large waste and even safety issues. The objective of control performance assessment is to develop tools that deliver information to users how well control-loop performances meet with their control targets. To reach this objective, one of the most important tasks is to find a benchmark against which the current control-loop performance can be assessed. The celebrated minimum variance control (MVC) benchmark introduced by Harris [3] is such a benchmark to assess the control-loop performance.

Proportional-integral-derivative (PID) controllers are extensively used in industrial feedback control loops. Improper PID settings may result in sluggish or oscillatory loop responses, poor disturbance rejection ability, low robustness and even safety problems. Hence, finding a benchmark to assess the performance of PID control loops is very important for industrial practice. The existing performance indices for PID control loops can be classified into the following categories:

1) Performance in terms of stochastic disturbances attenuation. The performance indices in this category are mainly based on the same idea of the MVC benchmark, with a specific consideration of the restricted PID controller structure. Ko & Edgar [6] and Sendjaja & Kariwala [7] used an iterative method and the sums of squares programming, respectively, to calculate the best achievable minimum variance bound for PID controllers.

2) Performance in terms of deterministic disturbance rejection. Tan, Marquez, & Chen [11] took the minimum singular value of the integral gain matrix as the load disturbance rejection benchmark. Huagglund [2] and Visioli [14] used the area index and the idle index to tell whether a PID control loop is sluggish or oscillatory when the load disturbance presents. Veronesi & Visioli [13] proposed an index on the integrated absolute error (IAE) for step load disturbance for integral processes.

3) Performance in terms of setpoint tracking. The traditional performance indices include the overshooting, rising time, and settling time. Grimble [1] studied the benchmark based on the LQG cost function with consideration on the restricted structure imposed by PID controllers. Recently, people have been focusing on the IAE-based indices. Swanda & Seborg [10] and Huang & Jeng [4] estimated the lower IAE bound for PI/PID control loops from step response by simulations. Veronesi & Visioli [12] proposed two indices based on IAE from closed-loop step responses by following the Skogestad internal model control tuning rule.

This paper continues the studies in the third category. While the existing studies in this category are limited to closed-loop step responses, we would also like to consider other types of setpoint changes that appear even more often than step changes in industrial practice, in order to avoid adverse effects caused by abrupt step changes. To be specific, we will establish in theory the lower bounds
of IAEs, based on the widely-used internal model control (IMC) principle, from closed-loop responses subject to step, ramp and other general types of setpoint changes. The lower bound is shown to be proportional to the process time delay, the desired closed-loop time constant, and the setpoint variation. By taking the lower bound of the IAE as a benchmark, we will propose an IMC-IAE-based index to assess the performance of PID control loops in terms of setpoint tracking.

This article is organized as follows: Section 2 reviews the IMC tuning rule briefly. Section 3 establishes the lower bound of the IAE based on the IMC principle, and proposes an IMC-IAE-based performance index. Numerical examples, experimental examples and an industrial case study are provided to validate the performance benchmark, and to illustrate the effectiveness of the proposed performance index in Sections 4, 5 and 6, respectively. Some concluding remarks are given at Section 7.

2. IMC TUNING RULE FOR PID CONTROLLERS

The PID controller based on the IMC tuning rule usually leads to a control loop with a good balance among setpoint tracking, disturbance rejection and robustness, and has been widely adopted in industrial practice for years. In this section, the IMC tuning rule for PID controllers is briefly reviewed.

\[ r(t) \rightarrow e(t) \rightarrow C(s) \rightarrow u(t) \rightarrow P(s) \rightarrow y(t) \]

Fig. 1. A SISO feedback control loop

Consider a SISO feedback control loop depicted in Fig. 1. Here \( P(s) \) and \( C(s) \) are the process and the PID controller, respectively; \( r(t) \), \( u(t) \) and \( y(t) \) are the setpoint, the control signal, and the process output, respectively. In this context, \( P(s) \) is confined to be a process that is stable, without integrals and negative zeros; hence, the process can be approximated by a first-order plus dead time (FOPDT) model,

\[ P(s) = \frac{K}{T_1 s + 1} e^{-\theta s}, \quad (1) \]

or a second-order plus dead time (SOPDT) model,

\[ P(s) = \frac{K}{T_1 s^2 + T_2 s + 1} e^{-\theta s}. \quad (2) \]

As \( \theta \) is crucial to the subsequent performance index, it is worthy to mention that \( \theta \) is the time delay of the low-order model (1) or (2), instead of the time delay of the actual process. For instance, the positive zero of a process can be removed by lumping it into the time delay part of a low-order approximated model [8].

The PID controller \( C(s) \) takes a non-interactive formulation,

\[ C(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right). \quad (3) \]

The IMC tuning rule gives the controller setting for the FOPDT model as

\[ K_p = \frac{T_1}{K(\tau_c + \theta)}, \quad T_i = T_1, \quad T_d = 0, \]

and that for the SOPDT model as

\[ K_p = \frac{T_2}{K \tau_c (\tau_c + \theta)}, \quad T_i = T_2, \quad T_d = T_1 \frac{T_2}{T_1}. \quad (4) \]

By taking the IMC tuning rule, the desired closed-loop response will be

\[ G_{CL}(s) = \frac{1}{(\theta + \tau_c)s + e^{-\theta s}} \approx \frac{1}{\tau_c s + 1} e^{-\theta s}. \quad (5) \]

Here the user-selected parameter \( \tau_c \) stands for the desired closed-loop time constant.

3. IAE BENCHMARK BASED ON IMC TUNING RULE

This section derives the lower bound of the IAE for an IMC-based PID controller in the closed-loop response subject to ramp or other general types of setpoint changes. By taking the lower bound as a benchmark, an IMC-IAE-based index is proposed for performance assessment of PID control loops.

3.1 The lower bound of the IAE for ramp response

Consider a ramp setpoint,

\[ r(t) = \begin{cases} kt, & 0 \leq t < T, \\ kT, & T \leq t < \infty. \end{cases} \]

Here \( k \) is the slope of the ramp signal, and takes a positive real value for the time being (the case of \( k < 0 \) will be discussed later); \( T \) is the time duration for the ramp signal to increase into its final constant value. As shown in Fig. 2, \( r(t) \) can be decomposed into two parts, namely, \( r_1(t) \) and \( r_2(t) \), i.e.,

\[ r(t) = r_1(t) + r_2(t), \quad (6) \]

where

\[ r_1(t) = kt, \quad 0 \leq t < \infty, \]

\[ r_2(t) = \begin{cases} 0, & 0 \leq t < T, \\ -k(t - T), & T \leq t < \infty. \end{cases} \]

The Laplace transform of \( r(t) \) is

\[ R(s) = R_1(s) + R_2(s) = \frac{k}{s^2} + \frac{-k}{s^2} e^{-T s}, \]

from which, together with (5), we reach the Laplace transform of the desired closed-loop response,

\[ Y(s) = G_{CL}(s)R(s) = \frac{k}{\tau_c} \left( \frac{e^{-\theta s}}{s^2(s + \frac{1}{\tau_c})} - \frac{e^{-(\theta + T)s}}{s^2(s + \frac{1}{\tau_c})} \right). \]

The inverse Laplace transform of \( Y(s) \) is,

\[ y(t) = \begin{cases} 0, & 0 \leq t < \theta , \\ k \left( \tau_c e^{-\frac{t - \theta}{\tau_c}} + t - \theta - \tau_c \right), & \theta \leq t < \theta + T, \\ k \left( \tau_c e^{-\frac{t - \theta}{\tau_c}} - \tau_c e^{-\frac{t - \theta - T}{\tau_c}} + T \right), & \theta + T \leq t < \infty. \end{cases} \quad (7) \]

From (6) and (7), the error signal \( e(t) := r(t) - y(t) \) and the lower bound of the associated IAE are calculated as follows. If \( \theta \leq T \), then
Under the IMC tuning rule, the desired closed-loop response will have no overshooting, i.e., $e(t) \geq 0$, $\forall t \geq 0$; thus, the lower bound of the corresponding IAE is

$$
IAE_{0, \theta < T} = \int_0^\infty |e(t)|dt
$$

$$
e(t) = \begin{cases}
kt, & 0 \leq t < \theta, \\
k(-\tau_e e^{-\frac{t-\theta}{\tau_c}} + \theta + \tau_c), & \theta \leq t < T, \\
k(-\tau_e e^{-\frac{T-t-\theta}{\tau_c} + \tau_c + \tau_c t - t}), & T \leq t < \theta + T, \\
k(-\tau_e e^{-\frac{T-t-\theta}{\tau_c} + \tau_c e^{-\frac{t-\theta}{\tau_c}}}), & \theta + T \leq t < \infty.
\end{cases}
$$

Fig. 2. The ramp setpoint $r(t)$ and its decomposition consisting of $r_1(t)$ and $r_2(t)$

The lower bound of the associated IAE is

$$
IAE_{0, \theta > T} = \int_0^\infty |e(t)|dt
$$

$$
e(t) = \begin{cases}
kt, & 0 \leq t < T, \\
k(T - kT e^{-\frac{t}{\tau_c}} + k(\theta + \tau_c - t), & T \leq t < \theta, \\
k(-kT e^{-\frac{t}{\tau_c}} + kT e^{-\frac{T-t-\theta}{\tau_c}}), & \theta \leq t < \theta + T, \\
k(-kT e^{-\frac{T-t-\theta}{\tau_c} + \tau_c e^{-\frac{t-\theta}{\tau_c}}}), & \theta + T \leq t < \infty.
\end{cases}
$$

Eqs. (8) and (9) are based on the assumption that the slope $k$ is positive; if $k < 0$, by following a similar derivation, the counterpart of (8) and (9) is $IAE_0 = -kT(\theta + \tau_c)$. Hence, in general, the lower bound of the IAE is

$$
IAE_0 = \int_0^\infty |e(t)|dt = |kT| (\theta + \tau_c).
$$

3.2 The lower bound of the IAE for a general-type response

Suppose that the setpoint $r(t)$ changes from one steady value to another, by following a general-type path such as that shown in Fig. 3. The setpoint can be approximated by a series of ramp signals

$$
r(t) \approx r_1(t) + r_2(t) + \ldots + r_N(t),
$$

where

$$
r_i(t) = \begin{cases}
0, & 0 \leq t < t_{i-1}, \\
k_i(t - t_{i-1}), & t_{i-1} \leq t < t_i, \\
k_i(t_i - t_{i-1}), & t_i \leq t < \infty.
\end{cases}
$$

Here $k_i$ is the slope of $r_i(t)$; $r(t_0)$ and $r(t_N)$ are the initial and final steady values of $r(t)$, respectively, i.e., $r(t) = r(t_0)$ for $t \leq t_0$ and $r(t) = r(t_N)$ for $t \geq t_N$. From (10) and the superposition principle (the closed-loop system is linear time-invariant), it is ready to obtain the lower bound of the IAE as

$$
IAE_0[r(t)] \approx IAE_0[r_1(t) + \ldots + r_N(t)]
$$

$$
\leq IAE_0[r_1(t)] + \ldots + IAE_0[r_N(t)]
$$

$$
= \sum_{i=1}^{\infty} (|k_1(t_1 - t_0)| + \ldots + |k_N(T_N - t_{N-1})|) (\theta + \tau_c).
$$

Here $IAE_0[r_i(t)]$ is the lower bound of the IAE for the $i$-th ramp signal $r_i(t)$. The equality in (12) holds under the conditions that all slopes of $r_i(t)$’s $(i = 1, \ldots, N)$ have the same sign, or that $r(t)$ keeps invariant for a sufficient period of time for the closed-loop system to arrive at a steady state before changing into the opposite direction. Under these conditions, when the duration of each $r_i(t)$ becomes shorter, the approximation error of $r(t)$ in (11) is getting smaller; therefore, the lower bound of the IAE is close to

$$
IAE_0 = \sum_{i=1}^{\infty} |r(t_{i+1}) - r(t_i)| (\theta + \tau_c).
$$

Obviously, (10) for ramp response is one special case of (13). So is the lower bound for step response, which has been proposed by Veronesi & Visioli [12].
3.3 IMC-IAE-based index

Based on the lower bound $IAE_0$ in (13), a dimensionless index is defined as the ratio between $IAE_0$ and the actual IAE from closed-loop data under step, ramp or other types of setpoint changes,

$$\eta_{IAE} = \frac{\min (IAE_0, IAE_{Actual})}{\max (IAE_0, IAE_{Actual})},$$

where $IAE_{Actual} = \sum_{i=1}^{\infty} |e(t_{i+1}) - e(t_i)|T_s$ for $T_s$ being the sampling period. Henceforth, it is referred to the IMC-IAE-based index.

Given collected data of $y(t)$ and $r(t)$ (see Fig. 1), the calculation of $\eta_{IAE}$ requires the determination of the time delay $\theta$ and the desired closed-loop time constant $\tau_c$.

- The time delay estimation has been an interesting research topic for years in various areas; see a recent survey in [9]. However, most of the existing methods have their own limitations; in particular, extra experiments are usually required to introduce special signals such as white noise to excite the unknown process. By contrast, it is desired for industrial practice to estimate $\theta$ based on the closed-loop step, ramp or some other simple response, without introducing extra experiments.

- The appearance of $\tau_c$ makes $\eta_{IAE}$ a user specified benchmark, which is indeed an advantage comparing to the MVC benchmark. In other words, users can determine $\tau_c$ as the desired closed-loop time constant, and evaluate the performance of the current control loop against the desired one. However, an improper selection of $\tau_c$ could make $\eta_{IAE}$ too large or too small, leading to erroneous conclusion on the control-loop performance. A fair selection of $\tau_c$ is to take the current closed-loop-time constant (to be estimated based on the collected data) as $\tau_c$. If $\eta_{IAE} \rightarrow 1$, the current control-loop performance is close to the expected by using the IMC-tuning rule. Note that $\eta_{IAE} = 1$ is achievable in practice, while the MVC-based index for PID control loops is usually quite far away from 1 even though the control-loop performance is satisfactory.

Another practical issue is about the noise effect on $\eta_{IAE}$. First, the noise affects the estimates of $\tau_c$ and $\theta$, whose accuracies are up to the open- and closed-loop identification techniques. Thus, the quality control of the two estimates are out of context here. Second, the noise affects also the calculation of the actual IAE, and may result in an incorrect estimate of $\eta_{IAE}$, despite a fact that the summation in (14) may enable $\eta_{IAE}$ somehow robust to noise. To resolve this issue, the noise-free closed-loop response $\hat{y}(t)$ is obtained, i.e.,

$$\hat{y}(s) = \frac{\hat{P}(s)C(s)}{1 + \hat{P}(s)C(s)}R(s).$$

Based on $\hat{y}(t)$ and $r(t)$, a noise-free estimate of $\eta_{IAE}$ can be calculated.

4. SIMULATION EXAMPLES

In this section, two simulation examples are presented to validate the lower bound of the IAE. The process in Fig. 1 is

$$P(s) = \frac{2.5}{5s^2 + 6.25s + 1}e^{-2s},$$

and the PID controller $C(s)$ follows the IMC tuning rule in (4). The setpoint is a ramp signal,

$$r(t) = \begin{cases} 
0, & 0 \leq t < 10, \\
(t - 10)/10, & 10 \leq t < 100, \\
90, & 100 \leq t < \infty.
\end{cases}$$

To validate the lower bound of the IAE, $IAE_0$ in (10), we vary the value of the desired closed-loop time constant as an integer multiple of the time delay $\theta$, i.e., $\tau_c = \lambda\theta$. Table 1 compares the theoretical lower bound of the IAE in (10) and the actual IAE. Here the actual IAE is calculated as that in (14) with the sampling period $T_s = 0.1$ sec. The two values are very close to each other for different values of $\tau_c$. The minor difference between the actual and theoretical values is due to the first-order approximation for the time delay in (5).

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$K_p$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$IAE_0$</th>
<th>$IAE_{Actual}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.625</td>
<td>5</td>
<td>0.8</td>
<td>360</td>
<td>365.75</td>
</tr>
<tr>
<td>2</td>
<td>0.417</td>
<td>5</td>
<td>0.8</td>
<td>540</td>
<td>544.50</td>
</tr>
<tr>
<td>3</td>
<td>0.312</td>
<td>5</td>
<td>0.8</td>
<td>720</td>
<td>724.50</td>
</tr>
<tr>
<td>4</td>
<td>0.265</td>
<td>5</td>
<td>0.8</td>
<td>900</td>
<td>904.50</td>
</tr>
<tr>
<td>5</td>
<td>0.208</td>
<td>5</td>
<td>0.8</td>
<td>1080</td>
<td>1084.50</td>
</tr>
</tbody>
</table>

To validate (13), we let the setpoint experience several continuous changes,

$$r(t) = \begin{cases} 
0, & 0 \leq t < 10, \\
(t - 10)/10, & 10 \leq t < 60, \\
5 + 5\sin \frac{(t - 60)\pi}{80}, & 60 \leq t < 100, \\
9 + e^{(t-100)/50}, & 100 \leq t < 150, \\
32.14, & 151 \leq t < \infty.
\end{cases}$$

Table 2 is the counterpart of Table 1 for this more general setpoint signal. The theoretical lower bounds of the IAE in (13) fit with the actual IAEs very well for different values of $\tau_c$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$K_p$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$IAE_0$</th>
<th>$IAE_{Actual}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.625</td>
<td>5</td>
<td>0.8</td>
<td>128.56</td>
<td>131.71</td>
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<tr>
<td>2</td>
<td>0.417</td>
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<td>0.8</td>
<td>192.84</td>
<td>194.45</td>
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<tr>
<td>3</td>
<td>0.312</td>
<td>5</td>
<td>0.8</td>
<td>257.12</td>
<td>258.73</td>
</tr>
<tr>
<td>4</td>
<td>0.265</td>
<td>5</td>
<td>0.8</td>
<td>321.41</td>
<td>323.01</td>
</tr>
<tr>
<td>5</td>
<td>0.208</td>
<td>5</td>
<td>0.8</td>
<td>385.69</td>
<td>387.30</td>
</tr>
</tbody>
</table>

5. EXPERIMENTAL EXAMPLES

This section provides experimental examples to illustrate the procedure of assessing the performance of a PID control loop using the proposed IMC-IAE-based index.
In the experiments, the process in Fig. 1 is a water tank system, whose cross-sectional area is about 320 cm². The water level of the tank is selected as the process variable (PV), with the range [0, 100]. The opening of the outlet valve is fixed, while the input valve is driven by a frequency convertor to control the inlet flow, i.e., the frequency of the convertor is the manipulated variable. The PID controller is in the non-interactive form as that in (3). The sampling period is 0.5 sec. Two experiments are performed for two different sets of PID controller parameters. In both experiments, the setpoint has initially been staying at the value 20 for a sufficient long time for the PV to initiate at the steady state. The setpoint experiences a ramp change as

\[ r(t) = \begin{cases} 
20 + 0.8t, & 0 \leq t < 25, \\
40, & 25 \leq t < 250.
\end{cases} \tag{17} \]

Fig. 4. The ramp setpoint (dashed), the measured PV (solid) for \( K_p = 1.2 \) and \( T_i = 10 \) and \( T_d = 0 \), and the measured PV (dashed-dotted) for \( K_p = 2.0249 \), \( T_i = 233.94 \) and \( T_d = 0 \).

In the first experiment, the PID controller parameters are \( K_p = 1.2 \), \( T_i = 10 \) and \( T_d = 0 \), and the corresponding ramp response is shown in Fig. 4 (solid line). Based on this closed-loop ramp response, an FOPDT model for the open-loop process is estimated,

\[ \hat{P}(s) = \frac{6.6469}{241.3700 + 1} e^{-1.1122s}. \tag{19} \]

Analogously to the first experiment, the performance assessment results are obtained, as given in Table 3. Both \( \eta_{IAE,y}(t) \) and \( \eta_{IAE,\hat{y}}(t) \) say that the control-loop performance is excellent and is very close to the expected one by using the IMC-tuning rule. In fact, if the IMC-tuning rule is used based on the model in (19), the resulted controller parameters are \( K_p = 2.0494 \), \( T_i = 241.37 \) and \( T_d = 0 \), which are almost the same as the PID controller parameters used in the current experiment.

### Table 3. Performance assessment results for the experimental examples

<table>
<thead>
<tr>
<th>Data set</th>
<th>1st Experiment</th>
<th>2nd Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_c )</td>
<td>8.8796</td>
<td>19.6608</td>
</tr>
<tr>
<td>( \theta )</td>
<td>1.2143</td>
<td>1.1122</td>
</tr>
<tr>
<td>IAE ( y )</td>
<td>200.6780</td>
<td>354.3800</td>
</tr>
<tr>
<td>IAE ( \hat{y} )</td>
<td>742.3969</td>
<td>446.9704</td>
</tr>
<tr>
<td>IAE ( \hat{y} )</td>
<td>681.1547</td>
<td>364.5743</td>
</tr>
<tr>
<td>( \eta_{IAE,y}(t) )</td>
<td>0.2703</td>
<td>0.7928</td>
</tr>
<tr>
<td>( \eta_{IAE,\hat{y}}(t) )</td>
<td>0.2946</td>
<td>0.9720</td>
</tr>
</tbody>
</table>

6. INDUSTRIAL CASE STUDY

This section uses the IMC-IAE index to assess the performance of an industrial PID control loop, which is the forced-draft control system for a thermal power plant at Shandong Province, China. That is, \( r(t) \), \( u(t) \) and \( y(t) \) in Fig. 1 represent the air flow demand, the control command for two forced-draft fans, and the measured air flow, respectively. From the DCS database, 20000 data points are collected with the sampling period 2 sec, standing for the normal operation of the forced-draft control system in 11.1 hours on May 21, 2010. The performance assessment results are obtained, as given in Table 3. Both \( \eta_{IAE,y}(t) \) and \( \eta_{IAE,\hat{y}}(t) \) say that the control-loop performance is excellent and is very close to the expected one by using the IMC-tuning rule. In fact, if the IMC-tuning rule is used based on the model in (19), the resulted controller parameters are \( K_p = 2.0494 \), \( T_i = 241.37 \) and \( T_d = 0 \), which are almost the same as the PID controller parameters used in the current experiment.

### Table 4. Performance assessment results for the industrial case study

<table>
<thead>
<tr>
<th>Data set</th>
<th>([2500,3500])</th>
<th>([17500,19500])</th>
<th>([1,20000])</th>
</tr>
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<tbody>
<tr>
<td>( \tau_c )</td>
<td>35.4389</td>
<td>39.3052</td>
<td>46.1849</td>
</tr>
<tr>
<td>( \theta )</td>
<td>10.2040</td>
<td>6.3628</td>
<td>4.5918</td>
</tr>
<tr>
<td>IAE ( y )</td>
<td>13324</td>
<td>197.49</td>
<td>129527</td>
</tr>
<tr>
<td>IAE ( \hat{y} )</td>
<td>18233</td>
<td>339.21</td>
<td>369314</td>
</tr>
<tr>
<td>IAE ( \hat{y} )</td>
<td>13979</td>
<td>17980</td>
<td>143386</td>
</tr>
<tr>
<td>( \eta_{IAE,y}(t) )</td>
<td>0.7308</td>
<td>0.5819</td>
<td>0.4188</td>
</tr>
<tr>
<td>( \eta_{IAE,\hat{y}}(t) )</td>
<td>0.9050</td>
<td>0.9109</td>
<td>0.9638</td>
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</tbody>
</table>
Analogously to the experimental example in Section 5, the closed-loop time constant is estimated for each data set, by fitting an FOPDT model for the closed-loop system. Another FOPDT model is identified for each data set to capture the open-loop process dynamics. Taking the estimated closed-loop time constant as $\tau_c$, the time delay of the identified FOPDT model for open-loop process dynamics as $\theta$, the lower bound of IAE in (13) and the IMC-IAE-based index in (14) are calculated. Table 4 lists the estimated closed-loop time constant $\tau_c$, the time delay $\theta$, the lower bound of IAE ($\text{IAE}_0$), the actual IAE based on $y(t)$ ($\text{IAE}_{\hat{y}(t)}$), and the IMC-IAE-based index $\eta_{\text{IAE},\hat{y}(t)}$ for each data set. To avoid an incorrect estimate of $\eta_{\text{IAE}}$ due to the noise, the noise-free estimate $\hat{y}(t)$ of $y(t)$ is also obtained based on the identified open-loop process model, also plotted together with $y(t)$ in Fig. 5. The resulted IAE based on $\hat{y}(t)$ ($\text{IAE}_{\hat{y}(t)}$) and the associated IMC-IAE-based index $\eta_{\text{IAE},\hat{y}(t)}$ are also given in Table 4.

In Table 4, $\eta_{\text{IAE},\hat{y}(t)}$ varies quite a lot, from 0.4188 for the whole data set to 0.7308 for the data set in the range [2500, 3500], grading the control-loop performance into different levels. However, all values of the index $\eta_{\text{IAE},\hat{y}(t)}$ based on $\hat{y}(t)$ are very close to 1, saying that the control-loop performance is excellent and is very close to the expected one by using the IMC-tuning rule.

For industrial applications of the proposed performance assessment technique, a related important issue is to validate the conclusion of the performance assessment based on the collected data. To resolve this issue, we have proposed a series of specially-designed model validation tests. That is, if the identified closed-loop and open-loop models have successfully passed the model validation tests, then the conclusion of the performance assessment would be trustworthy. One of the validation tests is to see the fitness between $\hat{y}(t)$ and $y(t)$. For instance, $\hat{y}(t)$ captures the main characteristic of $y(t)$ very well in Fig. 5.

7. CONCLUSIONS

This paper established the lower bound of the IAE for PID controllers tuned by following the IMC principle, from closed-loop responses subject to step, ramp or other types of setpoint changes. Based on the lower bound, an IMC-IAE-based index was proposed in (14) to assess the performance of PID control loops. Numerical examples validated the obtained lower bound as the performance benchmark. Experimental examples and an industrial case study illustrated the effectiveness of the IMC-IAE-based index.

REFERENCES