SOME SPECIAL DECENTRALIZED CONTROL PROBLEMS IN CONTINUOUS-TIME INTERCONNECTED SYSTEMS

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In this paper, some decentralized control problems composed of two subsystems are addressed from a special perspective. First, it is pointed out that some subsystems must be unstable to stabilize the overall interconnected system in some special cases. Then, a special kind of decentralized control problem is studied. This kind of problem can be viewed as harmonic control among independent subsystems. Research results show that two unstable systems can generate a stable system through some effective cooperations. And a linear matrix inequality method is provided for decentralized controller design by using the parameter-dependent Lyapunov function method. Two examples are given to illustrate the results.

Keywords: Decentralized control; interconnected systems; parameter-dependent Lyapunov function (PDLF); linear matrix inequality (LMI).

1. Introduction

Decentralized control of large scale systems has been studied extensively in the past four decades [3, 8, 13, 14]. The main difficulty of solving the decentralized control problem comes from the fact that the feedback gain is subject to structural constraints. Such constraints are of the same nature as the static output ones, which can be viewed as a full state feedback with structural constraints that select only the measured states. Generally, in the existing decentralized control method of large scale systems, closed-loop subsystems are all required to be stable. Even at the beginning of the study of large scale system theory, some people thought that a large scale system is decentrally stabilizable under the controllability condition by strengthening the stability degree of subsystems. Reference 15 showed that this idea is wrong through an example, and because of the existence of decentralized fixed modes, some large scale systems cannot be decentrally stabilized at all. Under the stability of subsystems, the actions of interconnections are always ignored and even viewed as disadvantages. This kind of study is disadvantageous for the study of the actions of interconnections. As with the development of society, interconnections play more and more important roles in social systems, economic systems,
power systems and complex networks. However, little work has been done on the effects of interconnections in large scale systems to the authors’ knowledge. In fact, it is an interesting topic in complex systems. Some simple systems can generate a complex large scale system through interconnections. Recently, some applications of the small gain theorem were given in Ref. 4 to strengthen the robust stability of interconnected systems. The small gain theorem was also used in Ref. 13. The effects of nonlinear input and output coupling was studied in Ref. 5. We study decentralized control problems from a special viewpoint in this paper. It is well known that the design of a decentralized controller is hard and the canonical diagonal Lyapunov function method is very conservative. The recently developed PDLF method for robust stability analysis [1,9,10,12] can be used in decentralized control problems and the conservativeness from diagonal Lyapunov functions can be reduced to some degree. This paper will also test the effectiveness of this method on special decentralized control problems introduced here.

This paper mainly focuses on interconnected systems composed of two subsystems. The results can be generalized to multiple subsystem cases. The rest of this paper is organized as follows. In Sec. 2, by studying the structure of interconnections we point out that it is impossible to stabilize all subsystems and the whole system simultaneously by using decentralized controllers in some special cases, that is, some subsystems must be unstable to stabilize the overall interconnected system. This result shows that the stability of interconnected systems is not only dependent on the stability degree of subsystems in some cases, but is closely dependent on the interconnections. In addition, for the sake of studying the effects of interconnections, we study a special kind of decentralized control problem which can be viewed as a harmonic stability problem among independent subsystems. The results show that two unstable subsystems can generate a stable interconnected system. In Sec. 3, we present an LMI decentralized controller design method. Two examples are given to illustrate the results in Sec. 4. Examples show that the parameter-dependent Lyapunov method is suitable for the problems discussed in this paper. The last section concludes the paper.

Throughout this paper, det(·) denotes the determinant of the corresponding matrix. Stability of matrices and polynomials refers to Hurwitz stability. The superscript \( T \) means transpose for real matrices.

2. The Effects of Unstable Subsystems

In this paper, we mainly consider the following interconnected system composed of two subsystems:

\[
\begin{align*}
\dot{x}_1 &= A_1 x_1 + A_{12} x_2 + B_1 u_1, \\
\dot{x}_2 &= A_2 x_2 + A_{21} x_1 + B_2 u_2.
\end{align*}
\]

\( u_1 = K_1 x_1, \quad u_2 = K_2 x_2, \quad A_{12}, A_{21} \) are matrices with compatible dimensions. We say system (1) can be decentrally stabilizable, i.e. there exist \( K_1 \) and \( K_2 \) such that the
state matrix of the closed-loop system

\[ A_{cl} = \begin{pmatrix} A_1 + B_1 K_1 & A_{12} \\ A_{21} & A_2 + B_2 K_2 \end{pmatrix} \]

is Hurwitz stable.

First, we consider a simple example. In system (1) if

\[
A_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix}, \quad A_{21} = \begin{pmatrix} 0 & \beta \\ 0 & 0 \end{pmatrix},
\]

\[
A_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad B_1 = B_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix},
\]

\[
K_1 = -(k_1 \quad k_2), \quad K_2 = -(k_3 \quad k_4),
\]

then

\[
A_{cl} = \begin{pmatrix} 0 & 1 & 0 & \alpha \\ -k_1 & -k_2 & 0 & 0 \\ 0 & \beta & 0 & 1 \\ 0 & 0 & -k_3 & -k_4 \end{pmatrix}.
\]

Obviously, at this time, \( \det(sI - A_{cl}) = \) \((s^2 + k_2 s + k_1)(s^2 + k_4 s + k_3) - \alpha \beta k_1 k_3. \) For this simple case, one can obtain the following results easily:

(i) When \( \alpha \beta = 1, 0 \) is a fixed mode.
(ii) When \( \alpha \beta > 1, \) for any \( k_i > 0, i = 1, 2, 3, 4, \) i.e. \( A_1 + B_1 K_1 \) and \( A_2 + B_2 K_2 \) are stable, \( A_{cl} \) cannot be Hurwitz stable (the constant term of its characteristic polynomial is less than 0 at this time).
(iii) When \( \alpha \beta < 1, \) it is possible that the interconnected system and two subsystems can be stabilized simultaneously.

An example as above was also introduced in Ref. 13 to show the complexity in interconnected systems. According to the analysis above, we give a special structural property of interconnections under which subsystems and the overall system cannot be simultaneously stabilized.

**Theorem 1.** If the interconnected system in (1) satisfies the following:

(i) there exists \( A_{12}' \) such that \( A_{12} = A_{12}' A_2, \) and \( A_{12}' B_2 = 0, \)
(ii) there exists \( A_{21}' \) such that \( A_{21} = A_{21}' A_1, \) and \( A_{21}' B_1 = 0, \)

then when \( \det(I - A_{21}' A_{12}') < 0, \) there are no \( K_1 \) and \( K_2 \) such that \( A_1 + B_1 K_1, A_2 + B_2 K_2 \) and \( A_{cl} \) are Hurwitz stable simultaneously.

**Proof.** Computing the determinant of \( A_{cl}, \) one can get

\[
\det(A_{cl}) = \det(A_1 + B_1 K_1) \cdot \det(A_2 + B_2 K_2 - A_{12}' A_1 (A_1 + B_1 K_1)^{-1} A_{12}' A_2).
\]
Noticing conditions (i) and (ii),
\[ \det(A_{cl}) = \det(A_1 + B_1K_1) \times \det(A_2 + B_2K_2 - A'_{21}(A_1 + B_1K_1)(A_1 + B_1K_1)^{-1}A'_{12}(A_2 + B_2K_2)), \]
that is,
\[ \det(A_{cl}) = \det(A_1 + B_1K_1) \det(A_2 + B_2K_2) \det(I - A'_{21}A'_{12}). \]

When \( A_1 + B_1K_1, A_2 + B_2K_2 \) are Hurwitz stable and \( \det(I - A'_{21}A'_{12}) < 0 \), one gets that the constant term of the characteristic polynomial of \( A_{cl} \) is less than zero. Therefore, \( A_{cl} \) is unstable.

**Remark 1.** Obviously, under the conditions in Theorem 1, when \( \det(I - A'_{21}A'_{12}) = 0 \), 0 is a fixed mode. When \( \det(I - A'_{21}A'_{12}) > 0 \), it is possible that there exist \( K_1 \) and \( K_2 \) such that \( A_1 + B_1K_1, A_2 + B_2K_2 \) are Hurwitz stable simultaneously. And if \( A_{cl} \) is stable, one of \( A_1 + B_1K_1 \) and \( A_2 + B_2K_2 \) must be unstable when \( \det(I - A'_{21}A'_{12}) < 0 \). This also shows the complexity of interconnected large scale systems. For the study of the effects of interconnections in large scale systems, it is important to design decentralized controllers when some subsystems must be unstable. At this time, the interconnections play real roles for stability of large scale systems. In addition, we should also notice that the conditions in Theorem 1 are restrictive. These conditions are related to the computation of the determinant of \( I - A_{cl} \). From this theorem, we can imagine that there exist other cases in which some subsystems must be unstable to stabilize the overall system. How to generalize Theorem 1 is a problem worth further study.

**Remark 2.** Because of the existence of unstable systems under the stability of interconnected systems, the traditional structural disturbance [13], i.e. communication failure between subsystems, is not allowed. Robustness analysis is a basic problem in control theory. For the interconnected systems discussed above, we can analyze its robustness under parametric uncertainty based on the parameter dependent Lyapunov method introduced in Refs. 10 and 11.

**Corollary 1.** For any interconnected matrix \( A = \begin{pmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{pmatrix} \), if \( A_{12} \) and \( A_{21} \) can be written as \( A_{12} = A'_{12}A_2, A_{21} = A'_{21}A_1 \), and \( \det(I - A'_{21}A'_{12}) < 0 \), then \( A_1, A_2 \) and \( A \) cannot be stable simultaneously.

When \( A_1 \) and \( A_2 \) are nonsingular, we can directly take \( A'_{12} = A_{12}A_2^{-1} \) and \( A'_{21} = A_{21}A_1^{-1} \).

Obviously, the results above can be generalized to cases of multiple subsystems. For example, for an interconnected system composed of three subsystems, its
closed-loop system matrix is given by

\[ A_{cl} = \begin{pmatrix} A_1 + B_1K_1 & A_{12} & A_{13} \\ A_{21} & A_2 + B_2K_2 & A_{23} \\ A_{31} & A_{32} & A_3 + B_3K_3 \end{pmatrix}. \]

Let \( \bar{A}_1 = \begin{pmatrix} A_1 \\ A_{21} \\ A_3 \end{pmatrix}, \bar{B}_1 = \text{diag}(B_1, B_2), \bar{A}_{13} = \begin{pmatrix} A_{13} \\ A_{23} \end{pmatrix}, \bar{A}_{31} = \begin{pmatrix} A_{31} & A_{32} \end{pmatrix}. \) If the following conditions are satisfied:

(i) there exists \( A'_{13} \) such that \( \bar{A}_{13} = \bar{A}'_{13}A_3, \) and \( A'_{13}B_3 = 0, \)

(ii) there exists \( A'_{31} \) such that \( \bar{A}_{31} = \bar{A}'_{31}A_1, \) and \( A'_{31}\bar{B}_1 = 0, \)

then there are no \( K_1, K_2 \) and \( K_3 \) such that \( \bar{A}_1 + \bar{B}_1 \text{diag}(K_1, K_2), A_3 + B_3K_3 \) and \( A_{12} \) are stable simultaneously when \( \det(I - A'_{31}A'_{13}) < 0. \)

Similar to the case of decentralized state feedbacks, we can also study the problem of decentralized dynamic output feedback stabilization. Suppose the outputs of two subsystems in (1) are \( y_1 = C_1x_1 \) and \( y_2 = C_2x_2, \) respectively. The decentralized dynamic controllers are given as

\[ x_{k1} = A_{k1}x_{k1} + B_{k1}y_1, \quad x_{k2} = A_{k2}x_{k2} + B_{k2}y_2, \]

\[ u_1 = C_{k1}x_{k1} + D_{k1}y_1, \quad u_2 = C_{k2}x_{k2} + D_{k2}y_2. \]

Then the state matrix of the closed-loop system is

\[ \bar{A}_{cl} = \begin{pmatrix} \bar{A}_1 + \bar{B}_1\bar{K}_1\bar{C}_1 & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_2 + \bar{B}_2\bar{K}_2\bar{C}_2 \end{pmatrix}, \]

where

\[ \bar{A}_1 = \begin{pmatrix} A_1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \bar{B}_1 = \begin{pmatrix} B_1 \\ 0 \end{pmatrix}, \quad \bar{C}_1 = \begin{pmatrix} C_1 \\ 0 \end{pmatrix}, \quad \bar{A}_{12} = \begin{pmatrix} A_{12} & 0 \\ 0 & 0 \end{pmatrix}, \]

\[ \bar{K}_1 = \begin{pmatrix} D_{k1} \\ B_{k1} \end{pmatrix}, \quad \bar{A}_2 = \begin{pmatrix} A_2 & 0 \\ 0 & 0 \end{pmatrix}, \quad \bar{B}_2 = \begin{pmatrix} B_2 \\ 0 \end{pmatrix}, \quad \bar{C}_2 = \begin{pmatrix} C_2 \\ 0 \end{pmatrix}, \]

\[ \bar{A}_{21} = \begin{pmatrix} A_{21} & 0 \\ 0 & 0 \end{pmatrix}, \quad \bar{K}_2 = \begin{pmatrix} D_{k2} \\ B_{k2} \end{pmatrix}. \]

At this time, the problem of dynamic output feedback stabilization can be viewed as a special problem of static state feedback stabilization. Obviously, if \( A_{12} \) and \( A_{21} \) satisfy the conditions of Theorem 1, then there are no decentralized controllers (2), i.e. \( \bar{K}_1 \) and \( \bar{K}_2, \) such that \( \bar{A}_1 + \bar{B}_1\bar{F}_1\bar{C}_1, \bar{A}_2 + \bar{B}_2\bar{F}_2\bar{C}_2 \) and \( \bar{A}_{cl} \) are stable simultaneously.

The above results show that in some cases the stability of interconnected systems is closely dependent on the interconnections. In order to study the actions of the interconnections between subsystems further, we study a special kind of decentralized control problem which can be viewed as a harmonic stability problem of subsystems.
3. A Special Harmonic Control Problem

Consider the following interconnected system:

\[ \dot{x}_1 = A_1 x_1 + b_{12} u_{12}, \quad \dot{x}_2 = A_2 x_2 + b_{21} u_{21}, \tag{3} \]

where \( u_{12} = k_{12} x_2, u_{21} = k_{21} x_1, b_{12} \) and \( b_{21} \) are given real vectors. \( k_{12} \) and \( k_{21} \) are real row vectors to be designed. In system (3), there is information interchange between two subsystems. This means that two systems are cooperating. For this special decentralized control problem, one can get some simple result for its stabilizability with the following lemma.

**Lemma 1.** Given a real monic polynomial \( f(\lambda) \) with degree \( n \), \( f(\lambda) \) has no real root if and only if \( f(x) > 0, \forall x \in \mathbb{R} \).

One can prove this lemma easily by writing \( f(\lambda) \) as

\[ f(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2) \cdots (\lambda - \lambda_n), \]

where \( \lambda_1, \lambda_2, \ldots, \lambda_n \) are roots of \( f(\lambda) \).

**Theorem 2.** If \( (A_1, b_{12}), (A_2, b_{21}) \) are controllable, and \( a = \text{tr}(A_1) + \text{tr}(A_2) < 0 \), where \( \text{tr}(\cdot) \) denotes the trace of the corresponding matrix, then there exist \( k_{12} \) and \( k_{21} \) such that \( A_d = \begin{pmatrix} A_1 & b_{12} \\ k_{21} & A_2 \end{pmatrix} \) is Hurwitz stable.

**Proof.** Suppose \( (A_1, b_{12}), (A_2, b_{21}) \) are with the standard controllable model. Let the orders of \( A_1 \) and \( A_2 \) be \( n \) and \( m \), respectively. Set \( H_1(s) = k_{21} (sI - A_1)^{-1} b_{12}, H_2(s) = k_{12} (sI - A_2)^{-1} b_{21}, k_{12} = (\beta_0, \beta_1, \ldots, \beta_{m-1}), k_{21} = (\alpha_0, \alpha_1, \ldots, \alpha_{n-1}), d_1(s) = \det(sI - A_1), d_2(s) = \det(sI - A_2), k_1(s) = \alpha_0 + \alpha_1 s + \cdots + \alpha_{n-1} s^{n-1}, k_2(s) = \beta_0 + \beta_1 s + \cdots + \beta_{m-1} s^{m-1} \), then \( A_d \) is stable if and only if the feedback system shown in Fig. 1 is stable, i.e. the polynomial \( d_d(s) = d_1(s) d_2(s) - k_1(s) k_2(s) \) is stable. Obviously, \( d_d(s) \) is a monic polynomial and the coefficient of \( s^{n+m-1} \) in \( d_d(s) \) is \( -a = -\text{tr}(A_1) + \text{tr}(A_2) > 0 \). Let \( d_1(s) d_2(s) = s^{n+m} - a s^{n+m-1} + d_0(s) \), then the stability of \( d_d(s) \) is completely determined by \( d(s) = d_0(s) - k_1(s) k_2(s) \). When at least one of \( n \) and \( m \) is odd, one can choose \( d(s) \) arbitrarily such that \( d_d(s) \) is stable, and decompose \( d(s) - d_0(s) \) into the product of real polynomials \( k_1(s) \) and \( k_2(s) \). This means that we find real vectors \( k_{12} \) and \( k_{21} \) such that \( A_d \) is stable. When both \( n \) and \( m \) are even, the degrees of \( k_1(s) \) and \( k_2(s) \) are odd, but the degree of \( k_1(s) k_2(s) \) is even. At this time, one needs to choose \( d(s) \) suitably such that \( d_d(s) \) is stable and \( d(s) - d_0(s) \) has a real root in order to decompose \( d(s) - d_0(s) \) into the product of two real polynomials. In fact, this can be completed as follows. First, choose a real number \( x \) such that \( d_d(x) < 0 \) by adjusting the roots of \( d_d(s) \) in the left half plane with the constraint that the sum of all its roots is \( a \), and \( d_d(x) = x^{n+m} + a x^{n+m-1} - d_0(x) < 0 \) can be guaranteed by enlarging the imaginary parts of the roots of \( d_d(s) \). Then, one knows that \( d_d(s) = s^{n+m} + a s^{n+m-1} - d_0(s) \) has a real root from Lemma 1. Hence, one can decompose \( d_d(s) = s^{n+m} + a s^{n+m-1} - d_0(s) \) into the product of two real polynomials \( k_1(s) \) and \( k_2(s) \). This completes the proof. \( \square \)
Remark 3. From the proof of the theorem, we know that the eigenvalues of $A_{cl}$ can be assigned arbitrarily with the only constraint $a = \lambda_1 + \cdots + \lambda_{n+m}$ when one of $n$ and $m$ is odd. When $n$ and $m$ are even simultaneously, the eigenvalues of $A_{cl}$ can also be assigned properly with the constraints $a = \lambda_1 + \cdots + \lambda_{n+m}$ and $k_{12}, k_{21}$ being real vectors.

Remark 4. System (3) can be viewed as cooperative behavior between two subsystems. Two subsystems can be unstable themselves, but they can realize a stable system through intercrossed feedback. Subsystem does not use the information itself, but it uses the other subsystem’s information. That is, they can help with each other to attain some target. Of course, there may be self-feedback in subsystem. We can imagine that under cooperations, subsystems need not be controllable or stabilizable themselves (see the following system).

If there is self-feedback in subsystem, system (3) can be stated as follows:

$$
\dot{x}_1 = A_1 x_1 + b_1 u_1 + b_{12} u_{12}, \quad \dot{x}_2 = A_2 x_2 + b_2 u_2 + b_{21} u_{21},
$$

where $u_{12}, u_{21}, b_{12}, b_{21}$ are given as in system (3), $b_1$ and $b_2$ are real vectors with compatible dimensions, $u_1 = k_1 x_1$ and $u_2 = k_2 x_2$. By using Theorem 2, one can get the following result easily.

Theorem 3. If $(A_1, [b_1,b_{12}])$, $(A_2, [b_2,b_{21}])$ are controllable and $b_1, b_{12}$ are not zero vectors simultaneously, then there are real vectors $k_1, k_{12}, k_2, k_{21}$ such that system (4) is stable, i.e. $A_{cl} = \begin{pmatrix}
A_1 + b_1 k_1 & b_{12} k_{12} \\
b_{21} k_{21} & A_2 + b_2 k_2
\end{pmatrix}$ is Hurwitz stable.

Remark 5. One can see clearly in Theorem 3, $(A_1, b_1)$ and $(A_2, b_2)$ need not be controllable or stabilizable. Theorem 4 shows that two subsystems with effective control can cooperate easily for the sake of stability. The actions of interconnections are shown here to some degree. And obviously, the framework in Theorems 1, 2 and 3 can be generalized to the multi-subsystem cases.

4. Parameter-Dependent Lyapunov Method

Although we have discussed some special decentralized control problems in the sections above, it is still hard to design decentralized controllers. In what follows, we introduce the parameter-dependent Lyapunov method. By using this method we present an LMI-based [2, 7] design method for the problems discussed above. First, we introduce the following lemma to begin this section.
Lemma 2. Given a real matrix $A \in \mathbb{R}^{n \times n}$, $A$ is Hurwitz stable if, and only if, there exist a matrix $P = P^T > 0$ and any matrix $V$ such that

$$
\begin{pmatrix}
-V - V^T & V^T A^T + P & V^T \\
AV + P & -P & 0 \\
V & 0 & -P
\end{pmatrix} < 0.
$$

(5)

One can turn (5) into the Lyapunov inequality $A^T P + PA < 0$ easily by using the well-known projection lemma in the LMI method. By introducing a new variable $V$, the products of $PA$ and $A^T P$ are relaxed to new products $AV$ and $V^T A^T$. $V$ need not be symmetric and positive definite. In this way Lyapunov matrix $P$ can be parameter-dependent for the study of robust stability and robust performances [10, 11]. The case of the diagonal blocked matrix $V$ for decentralized control of discrete-time systems was considered in Ref. 11. Here, we discuss the upper trigonal constraint of $V$ for system (1) as follows; the lower trigonal constraint can be considered similarly. Sometimes, the upper trigonal constraint is less conservative than the diagonal constraint. Corresponding to system (1), we suppose that

$$
V = \begin{pmatrix} V_1 & \lambda V_1 \\ 0 & V_2 \end{pmatrix}, \quad X = \begin{pmatrix} X_1 & \lambda X_1 \\ 0 & X_2 \end{pmatrix},
$$

(6)

where the dimensions of $V_1$ and $V_2$ are identical to those of $A_1$ and $A_2$, the dimensions of $X_1$ and $X_2$ are identical to those of $K_1$ and $K_2$, respectively, and $\lambda$ is a parameter to be found. The parameter $\lambda$ can provide one more degree of freedom to reduce the conservativeness of the method above (see Ref. 6 for the similar method). For simplicity, we assume that $V$ and $X$ acquire the upper trigonal structure as in (6). In fact, it is only fit for the case of $\text{order}(A_1) = \text{order}(A_2)$. If $\text{order}(A_1) \neq \text{order}(A_2)$, for example, $\text{order}(A_1) < \text{order}(A_2)$, we can add zero blocks in $V$ and $X$ as follows to solve such cases:

$$
V = \begin{pmatrix} V_1 & V_{12} \\ 0 & V_2 \end{pmatrix}, \quad X = \begin{pmatrix} X_1 & X_{12} \\ 0 & X_2 \end{pmatrix},
$$

where $V_{12} = (\lambda V_1 \ 0)$, $X_{12} = (\lambda X_1 \ 0)$. When $\text{order}(A_1) > \text{order}(A_2)$, we assume $V$ and $X$ acquire the following lower trigonal structure

$$
V = \begin{pmatrix} V_1 & 0 \\ V_{21} & V_2 \end{pmatrix}, \quad X = \begin{pmatrix} X_1 & 0 \\ X_{21} & X_2 \end{pmatrix},
$$

where $V_{21} = (\lambda V_2 \ 0)$, $X_{21} = (\lambda X_2 \ 0)$, then we can get the similar result as in the following theorem.

Let

$$
A = \begin{pmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{pmatrix}, \quad B = \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix}, \quad K = \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix}
$$

(7)

in system (1). By Lemma 2, one can get the following result for stability of system (1).
Theorem 4. If there are $P = P^T$ and $V, X$ with the form of (6) such that

$$
\begin{pmatrix}
-V - V^T & V^T A^T + X^T B^T + P & V^T \\
AV + BX + P & -P & 0 \\
V & 0 & -P
\end{pmatrix} < 0,
$$

then there exists a diagonal blocked matrix $K$ as in (7) such that $A_{cl} = A + BK$ is stable. At this time, decentralized controllers can be obtained as $K_1 = X_1 V_1^{-1}, K_2 = X_2 V_2^{-1}$.

Remark 6. From Theorem 4, one can see that $P$ is not blocked, and $V_1, V_2$ are generally not symmetric — and of course not positive definite. Intuitively, one can imagine that at this time, $A_1 + B_1 K_1$ or $A_2 + B_2 K_2$ can be unstable under stability of $A_{cl}$. One can also see this from the following examples. In addition, one can establish some similar results for systems (3) and (4).

5. Examples

Example 1. In the simple example studied in Sec. 2, let $\alpha = \beta = 2$. From Theorem 4 ($\lambda = 1$), one can get a $K = \begin{pmatrix} -2.62 & -5.69 & 0 & 0 \\ 0 & 0 & 1.13 & -11.48 \end{pmatrix}$. Obviously $A_2 + B_2 K_2$ is not stable here. Using the method in Theorem 4 ($\lambda = 1$), one can get decentralized controllers $k_{21} = (-1.7976, -3.0015, -5.4645, -4.6861)$, $k_{12} = (0.1507, 1.0410, 0.8293, 1.3124)$ such that $A_{cl}$ is stable in (3).

From the examples above, one can see that subsystems need not be stable; even in some special cases, some subsystems must be unstable. This shows the special effects of interconnections.
6. Conclusion

This paper is devoted to studying decentralized control of some special interconnected systems. Some simple interconnected structures are established in which subsystems need not be stable. The results here can be generalized to cases of multiple subsystems. An LMI-based decentralized controller design method is also given by using the parameter-dependent Lyapunov function method. This method is suitable for the problems here in some cases, but is still very conservative. It is interesting to develop a more effective decentralized controller design method for the problems here. We hope that these results can be helpful for understanding the actions of interconnections in large scale systems.

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References